Foundations of Artificial Intelligence

16. State-Space Search: Greedy BFS, A*, Weighted A*

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State-Space Search: Overview

Chapter overview: state-space search

- 5.–7. Foundations
- 8.–12. Basic Algorithms
- 13.–19. Heuristic Algorithms
  - 13. Heuristics
  - 14. Analysis of Heuristics
  - 15. Best-first Graph Search
  - 16. Greedy Best-first Search, A*, Weighted A*
  - 17. IDA*
  - 18. Properties of A*, Part I
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16.1 Introduction

What Is It About?

In this chapter we study last chapter's algorithms in more detail:

- greedy best-first search
- A^*
- weighted A^*

16.2 Greedy Best-first Search

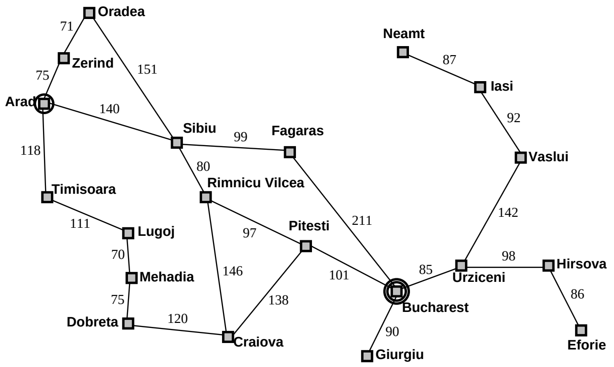
Greedy Best-first Search

Greedy Best-first Search

only consider the heuristic: $f(n) = h(n.\text{state})$

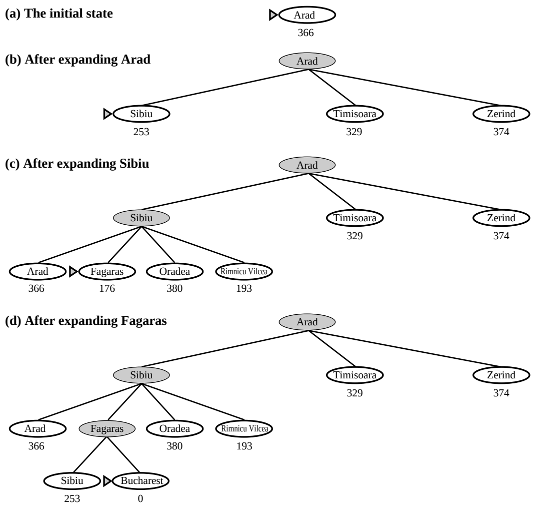
Note: usually without reopening (for reasons of efficiency)

Example: Greedy Best-first Search for Route Planning



Arad	366
Bucharest	0
Craiova	160
Drobeta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	100
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Example: Greedy Best-first Search for Route Planning



Greedy Best-first Search: Properties

- complete with safe heuristics (like all variants of best-first graph search)
- suboptimal: solutions can be arbitrarily bad
- often very fast: one of the fastest search algorithms in practice
- monotonic transformations of h (e.g. scaling, additive constants) do not affect behaviour (Why is this interesting?)

16.3 A*

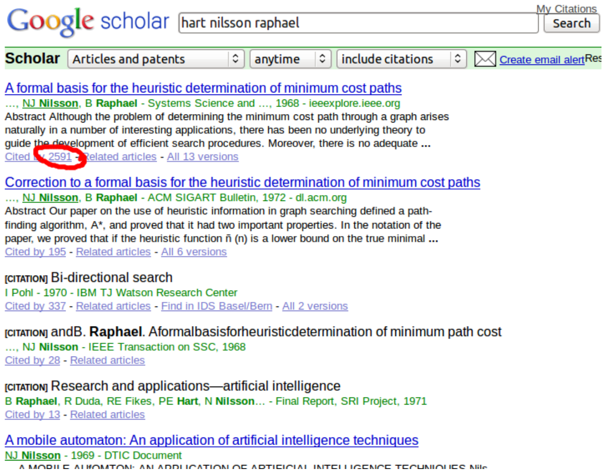
A*

A*

combine greedy best-first search with uniform cost search:
$f(n) = g(n) + h(n.\text{state})$

- trade-off between path cost and proximity to goal
- $f(n)$ estimates overall cost of cheapest solution from initial state via n to the goal

A*: Citations



hart nilsson raphael

Scholar | Articles and patents | anytime | include citations

A formal basis for the heuristic determination of minimum cost paths
…, NJ **Nilsson**, B **Raphael** - Systems Science and …, 1968 - ieeexplore.ieee.org
Abstract Although the problem of determining the minimum cost path through a graph arises naturally in a number of interesting applications, there has been no underlying theory to guide the development of efficient search procedures. Moreover, there is no adequate **...**
Cited by 2591 - Related articles - All 13 versions

Correction to a formal basis for the heuristic determination of minimum cost paths
…, NJ **Nilsson**, B **Raphael** - ACM SIGART Bulletin, 1972 - dl.acm.org
Abstract Our paper on the use of heuristic information in graph searching defined a path-finding algorithm, A*, and proved that it had two important properties. In the notation of the paper, we proved that if the heuristic function ñ (n) is a lower bound on the true minimal **...**
Cited by 195 - Related articles - All 6 versions

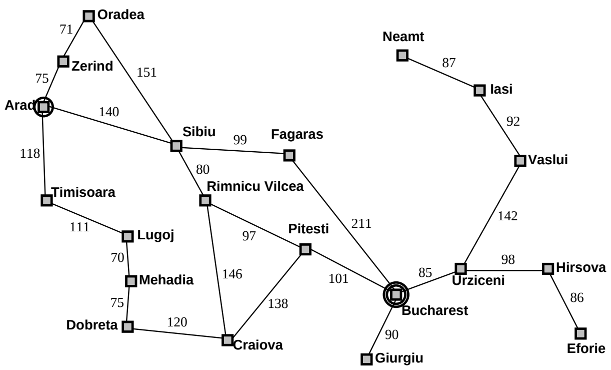
[CITATION] Bi-directional search
I Pohl - 1970 - IBM TJ Watson Research Center
Cited by 337 - Related articles - Find in IDS Basel/Bern - All 2 versions

[CITATION] andB. **Raphael**. Aformalbasisforheuristicdetermination of minimum path cost
…, NJ **Nilsson** - IEEE Transaction on SSC, 1968
Cited by 28 - Related articles

[CITATION] Research and applications—artificial intelligence
B **Raphael**, R Duda, RE Fikes, PE **Hart**, N **Nilsson**… - Final Report, SRI Project, 1971
Cited by 13 - Related articles

A mobile automaton: An application of artificial intelligence techniques
NJ **Nilsson** - 1969 - DTIC Document
A MOBILE AUTOMTON: AN APPLICATION OF ARTIFICIAL INTELLIGENCE TECHNIQUES Nils

Example: A* for Route Planning



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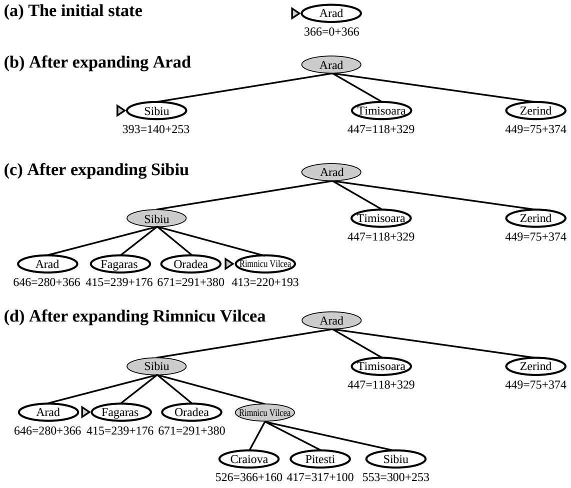
Example: A* for Route Planning



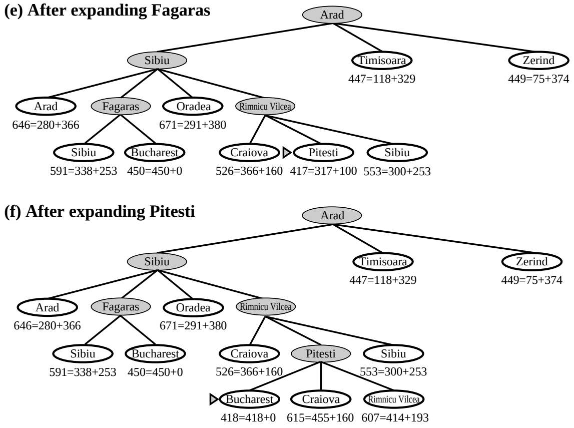
(a) The initial state

(b) After expanding Arad

(c) After expanding Sibiu

(d) After expanding Rimnicu Vilcea

Example: A* for Route Planning



(e) After expanding Fagaras

(f) After expanding Pitesti

A*: Properties

- complete with safe heuristics (like all variants of best-first graph search)
- with reopening: optimal with admissible heuristics
- without reopening: optimal with heuristics that are admissible and consistent

⇝ proofs: Chapters 18 and 19

A*: Implementation Aspects

some practical remarks on implementing A*:

- common bug: reopening not implemented although heuristic is not consistent
- common bug: duplicate test “too early” (upon generation of search nodes)
- common bug: goal test “too early” (upon generation of search nodes)
- all these bugs lead to loss of optimality and can remain undetected for a long time

16.4 Weighted A*

Weighted A*

Weighted A*

A* with more heavily weighted heuristic:
$f(n) = g(n) + w \cdot h(n.\mathrm{state})$,
where weight $w \in \mathbb{R}_0^+$ with $w \geq 1$ is a freely choosable parameter

Note: $w < 1$ is conceivable, but usually not a good idea
(Why not?)

Weighted A*: Properties

weight parameter controls “greediness” of search:

- $w = 0$: like uniform cost search
- $w = 1$: like A*
- $w \to \infty$: like greedy best-first search

with $w \geq 1$ properties analogous to A*:

- h admissible:
  found solution guaranteed to be at most w times as expensive as optimum when reopening is used
- h admissible and consistent:
  found solution guaranteed to be at most w times as expensive as optimum; no reopening needed

(without proof)

16.5 Summary

Summary

best-first graph search with evaluation function f:

- $f = h$: greedy best-first search
  suboptimal, often very fast
- $f = g + h$: A*
  optimal if h admissible and consistent
  or if h admissible and reopening is used
- $f = g + w \cdot h$: weighted A*
  for $w \geq 1$ suboptimality factor at most w
  under same conditions as for optimality of A*