

# Foundations of Artificial Intelligence

## 12. State-Space Search: Depth-first Search & Iterative Deepening

Malte Helmert and Gabriele Röger

University of Basel

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March 20, 2017 — 12. State-Space Search: Depth-first Search & Iterative Deepening

## 12.1 Depth-first Search

## 12.2 Iterative Deepening

## 12.3 Summary

## State-Space Search: Overview

### Chapter overview: state-space search

- ▶ 5.–7. Foundations
- ▶ 8.–12. Basic Algorithms
  - ▶ 8. Data Structures for Search Algorithms
  - ▶ 9. Tree Search and Graph Search
  - ▶ 10. Breadth-first Search
  - ▶ 11. Uniform Cost Search
  - ▶ 12. Depth-first Search and Iterative Deepening
- ▶ 13.–19. Heuristic Algorithms

## 12.1 Depth-first Search

## Depth-first Search

Depth-first search (DFS) expands nodes in opposite order of generation (LIFO).

↪ deepest node expanded first

↪ open list implemented as stack

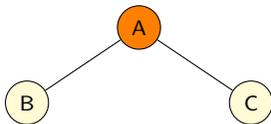
German: Tiefensuche

## Depth-first Search: Example



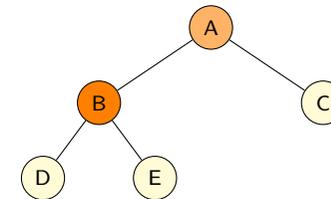
open: A

## Depth-first Search: Example



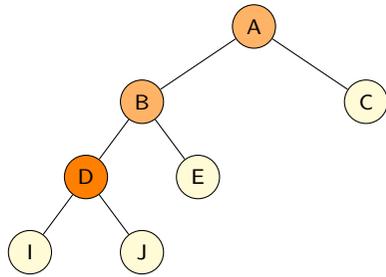
open: C, B

## Depth-first Search: Example



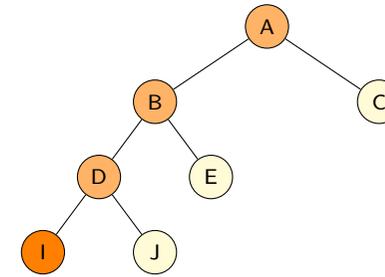
open: C, E, D

## Depth-first Search: Example



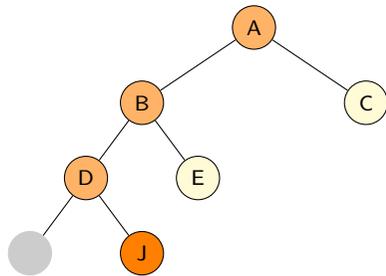
*open:* C, E, J, I

## Depth-first Search: Example



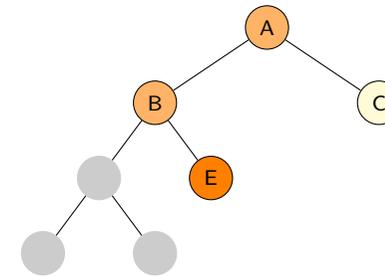
*open:* C, E, J

## Depth-first Search: Example



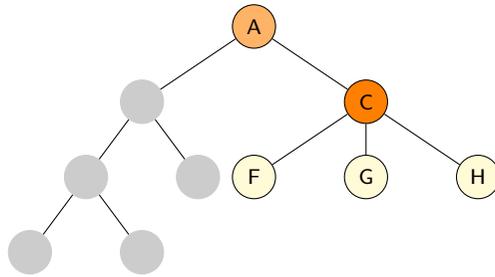
*open:* C, E

## Depth-first Search: Example



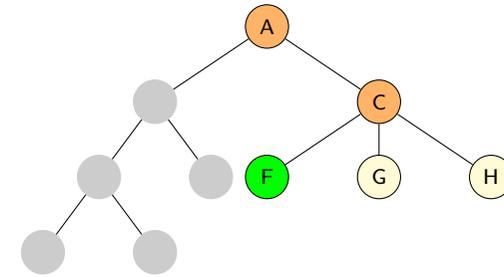
*open:* C

## Depth-first Search: Example



open: H, G, F

## Depth-first Search: Example



↔ solution found!

## Depth-first Search: Some Properties

- ▶ almost always implemented as a **tree search** (we will see why)
- ▶ **not complete, not semi-complete, not optimal** (Why?)
- ▶ complete for **acyclic** state spaces,  
e.g., if state space directed tree

## Reminder: Generic Tree Search Algorithm

reminder from Chapter 9:

## Generic Tree Search

```

open := new OpenList
open.insert(make_root_node())
while not open.is_empty():
    n := open.pop()
    if is_goal(n.state):
        return extract_path(n)
    for each ⟨a, s'⟩ ∈ succ(n.state):
        n' := make_node(n, a, s')
        open.insert(n')
return unsolvable

```

## Depth-first Search (Non-recursive Version)

depth-first search (non-recursive version):

Depth-first Search (Non-recursive Version)

```

open := new Stack
open.push_back(make_root_node())
while not open.is_empty():
    n := open.pop_back()
    if is_goal(n.state):
        return extract_path(n)
    for each ⟨a, s'⟩ ∈ succ(n.state):
        n' := make_node(n, a, s')
        open.push_back(n')
return unsolvable
  
```

## Non-recursive Depth-first Search: Discussion

discussion:

- ▶ there isn't much wrong with this pseudo-code (as long as we ensure to release nodes that are no longer required when using programming languages without garbage collection)
- ▶ however, depth-first search as a **recursive algorithm** is simpler and more efficient
- ↪ CPU stack as implicit open list
- ↪ no search node data structure needed

## Depth-first Search (Recursive Version)

```

function depth_first_search(s)
if is_goal(s):
    return ⟨⟩
for each ⟨a, s'⟩ ∈ succ(s):
    solution := depth_first_search(s')
    if solution ≠ none:
        solution.push_front(a)
        return solution
return none
  
```

main function:

Depth-first Search (Recursive Version)

```
return depth_first_search(init())
```

## Depth-first Search: Complexity

time complexity:

- ▶ If the state space includes paths of length  $m$ , depth-first search can generate  $O(b^m)$  nodes, even if much shorter solutions (e.g., of length 1) exist.
- ▶ On the other hand: in the **best case**, solutions of length  $\ell$  can be found with  $O(b\ell)$  generated nodes. (Why?)
- ▶ improvable to  $O(\ell)$  with **incremental successor generation**

space complexity:

- ▶ only need to store nodes **along currently explored path** ("along": nodes on path and their children)
- ↪ space complexity  $O(bm)$  if  $m$  maximal search depth reached
- ▶ low memory complexity main reason why depth-first search interesting despite its disadvantages

## 12.2 Iterative Deepening

## Depth-limited Search

### depth-limited search:

- ▶ depth-first search which **prunes** (does not expand) all nodes at a given depth  $d$

↔ not very useful on its own, but important ingredient of more useful algorithms

German: tiefenbeschränkte Suche

## Depth-limited Search: Pseudo-Code

```

function depth_limited_search( $s$ ,  $depth\_limit$ ):
  if is_goal( $s$ ):
    return  $\langle \rangle$ 
  if  $depth\_limit > 0$ :
    for each  $\langle a, s' \rangle \in succ(s)$ :
       $solution :=$  depth_limited_search( $s'$ ,  $depth\_limit - 1$ )
      if  $solution \neq \text{none}$ :
         $solution.push\_front(a)$ 
        return  $solution$ 
  return none

```

## Iterative Deepening Depth-first Search

### iterative deepening depth-first search (iterative deepening DFS):

- ▶ **idea**: perform a sequence of depth-limited searches with increasing depth limit
- ▶ sounds wasteful (each iteration repeats all the useful work of all previous iterations)
- ▶ in fact overhead acceptable (↔ analysis follows)

### Iterative Deepening DFS

```

for  $depth\_limit \in \{0, 1, 2, \dots\}$ :
   $solution :=$  depth_limited_search( $init()$ ),  $depth\_limit$ )
  if  $solution \neq \text{none}$ :
    return  $solution$ 

```

German: iterative Tiefensuche

## Iterative Deepening DFS: Properties

combines advantages of breadth-first and depth-first search:

- ▶ (almost) like BFS: **semi-complete** (however, not complete)
- ▶ like BFS: **optimal** if all actions have same cost
- ▶ like DFS: only need to store nodes along one path  
 $\rightsquigarrow$  space complexity  $O(bd)$ , where  $d$  minimal solution length
- ▶ time complexity only slightly higher than BFS  
 $(\rightsquigarrow$  analysis soon)

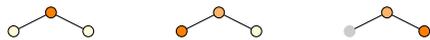
## Iterative Deepening DFS: Example

depth limit: 0



## Iterative Deepening DFS: Example

depth limit: 1

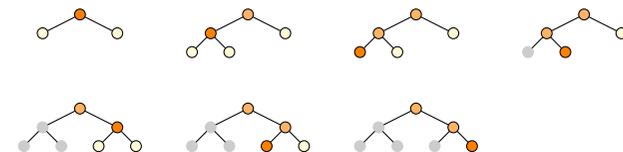


expansions in this round: 3

total expansions: 1 + 3

## Iterative Deepening DFS: Example

depth limit: 2

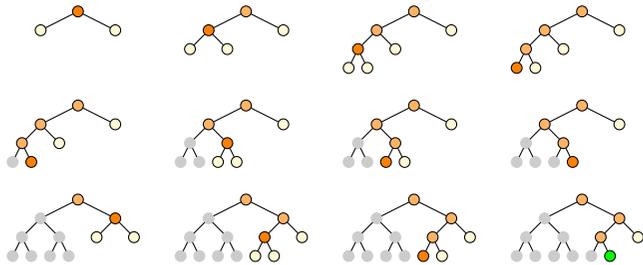


expansions in this round: 7

total expansions: 1 + 3 + 7

## Iterative Deepening DFS: Example

depth limit: 3



expansions in this round: 12

total expansions:  $1 + 3 + 7 + 12$

↪ solution found!

## Iterative Deepening DFS: Complexity Example

time complexity (generated nodes):

breadth-first search	$1 + b + b^2 + \dots + b^{d-1} + b^d$
iterative deepening DFS	$(d + 1) + db + (d - 1)b^2 + \dots + 2b^{d-1} + 1b^d$

example:  $b = 10, d = 5$

breadth-first search	$1 + 10 + 100 + 1000 + 10000 + 100000$ = 111111
iterative deepening DFS	$6 + 50 + 400 + 3000 + 20000 + 100000$ = 123456

for  $b = 10$ , only 11% more nodes than breadth-first search

## Iterative Deepening DFS: Time Complexity

Theorem (time complexity of iterative deepening DFS)

Let  $b$  be the branching factor and  $d$  be the minimal solution length of the given state space. Let  $b \geq 2$ .

Then the **time complexity** of iterative deepening DFS is

$$(d + 1) + db + (d - 1)b^2 + (d - 2)b^3 + \dots + 1b^d = O(b^d)$$

and the **memory complexity** is

$$O(bd).$$

## Iterative Deepening DFS: Evaluation

Iterative Deepening DFS: Evaluation

Iterative Deepening DFS is often the method of choice if

- ▶ **tree search is adequate** (no duplicate elimination necessary),
- ▶ all **action costs** are identical, and
- ▶ the **solution depth is unknown**.

## 12.3 Summary

## Summary

**depth-first search:** expand nodes in **LIFO** order

- ▶ usually as a **tree search**
- ▶ easy to implement **recursively**
- ▶ very **memory-efficient**
- ▶ can be combined with **iterative deepening** to combine many of the good aspects of breadth-first and depth-first search

## Comparison of Blind Search Algorithms

**completeness, optimality, time and space complexity**

criterion	search algorithm				
	breadth-first	uniform cost	depth-first	depth-limited	iterative deepening
complete?	yes*	yes	no	no	semi
optimal?	yes**	yes	no	no	yes**
time	$O(b^d)$	$O(b^{\lfloor c^*/\epsilon \rfloor + 1})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$
space	$O(b^d)$	$O(b^{\lfloor c^*/\epsilon \rfloor + 1})$	$O(bm)$	$O(b\ell)$	$O(bd)$

$b \geq 2$  branching factor  
 $d$  minimal solution depth  
 $m$  maximal search depth  
 $\ell$  depth limit  
 $c^*$  optimal solution cost  
 $\epsilon > 0$  minimal action cost

remarks:

\* for BFS-Tree: semi-complete  
 \*\* only with uniform action costs