

Foundations of Artificial Intelligence

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Exercise Sheet 9

Due: May 3, 2017

Exercise 9.1 (2+2 marks)

Compile the formulas below to CNF by applying the logical equivalences from the lecture (print version of Chapter 29, slide 24) and additionally commutativity (1) and associativity (2):

$$\varphi \wedge \psi \equiv \psi \wedge \varphi \text{ and } \varphi \vee \psi \equiv \psi \vee \varphi \quad (1)$$

$$((\varphi \wedge \psi) \wedge \eta) \equiv (\varphi \wedge (\psi \wedge \eta)) \text{ and } ((\varphi \vee \psi) \vee \eta) \equiv (\varphi \vee (\psi \vee \eta)). \quad (2)$$

Provide all intermediate formulas that result from applying an equivalence transformation. If using commutativity or associativity, you do not need to provide the steps.

(a) $(\neg P \vee Q) \rightarrow R$

(b) $(\neg(A \rightarrow B)) \vee (\neg C \wedge A)$

Exercise 9.2 (2+2 marks)

In the lecture, we have shown that the resolution method can be used for reasoning by a reduction to testing unsatisfiability. In this way, use the resolution method to show that the given formulas follow logically from the given set of formulas. Compare the number of required resolution steps with the number of entries that would have been necessary to show the same result with a truth table.

(a) Show that $K \vee I$ follows logically from $\{\neg R \vee K, R \vee \neg K, R \vee S, R \vee L, \neg S \vee L, \neg L \vee I\}$.

(b) Show that $\neg(\neg C \vee D)$ follows logically from $\{\{A, B, C\}, \{\neg A, \neg B, D\}, \{A, \neg B, C\}, \{B, C, D\}, \{\neg D, E\}, \{\neg D, \neg E\}\}$.

Exercise 9.3 (2+2 marks)

Use DPLL to show that the following sets of clauses are satisfiable or unsatisfiable. For the variable selection strategy, always choose a variable that occurs in the highest number of clauses. If using the splitting rule for some variable v , always consider the assignment $v \mapsto \mathbf{T}$ first. Provide the same intermediate results and applied rules of DPLL that were also given in the example in the lecture.

(a) $\{\{P, \neg Q\}, \{\neg P, Q\}, \{Q, \neg R\}, \{S\}, \{\neg S, \neg Q, \neg R\}, \{S, R\}\}$

(b) $\{\{P, Q, S, T\}, \{P, S, \neg T\}, \{Q, \neg S, T\}, \{P, \neg S, \neg T\}, \{P, \neg Q\}, \{\neg R, \neg P\}, \{R\}\}$

Important: The exercise sheets can be submitted in groups of two students. Please provide both student names on the submission.