Foundations of Artificial Intelligence

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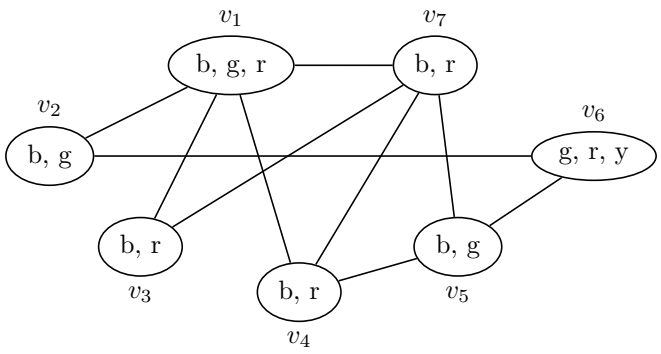
Exercise Sheet 7

Due: April 19, 2017

Exercise 7.1 (4 marks)

Consider the constraint network for the graph coloring problem that has been introduced on the lecture slides:



Provide the search tree that is created by applying naive backtracking on the depicted problem. Use the following static strategies on variable and value orderings:

- Variable ordering:
  (a) First select the variable with the smallest number of remaining values (*Minimum Remaining Values*)
  (b) Then break ties by selecting the variable that occurs in the largest number of non-trivial contraints (*Most Constrained Variable*)
  (c) If the choice is still not unique, break ties by selecting the variable with the smallest index.
- Value ordering: alphabetical (i.e., select b before g, g before r and r before y)

As on the lecture slides, your search tree should be complete and contain all solutions. Depict it in a similar style and discuss its size in comparison to the size of the search tree of the lecture slides.

Exercise 7.2 (1+3 marks)

Consider the 6 queens problem with the partial assignment $\alpha = \{v_1 \mapsto 2, v_2 \mapsto 4\}$:

	v_1	v_2	v_3	v_4	v_5	v_6
1						
2	q					
3						
4		q				
5						
6						

In the following, you may assume that the positions of the two queens that are already on the board are fixed, i.e., that the domain of the corresponding variables contains only the single entry that encodes the depicted position. The domain of the remaining variables contains all 6 possible values, though, which leads to the following domains for all variables:

$$\begin{aligned}
\mathrm{dom}(v_1) &= \{2\} \\
\mathrm{dom}(v_2) &= \{4\} \\
\mathrm{dom}(v_3) &= \{1,2,3,4,5,6\} \\
\mathrm{dom}(v_4) &= \{1,2,3,4,5,6\} \\
\mathrm{dom}(v_5) &= \{1,2,3,4,5,6\} \\
\mathrm{dom}(v_6) &= \{1,2,3,4,5,6\}
\end{aligned}$$

(a) Determine the domains of all variables after applying forward checking in α.

(b) Apply the AC-3 algorithm that has been presented in the lecture on the constraint network $\mathcal{C}$ with the domains that are the result of (a) until arc consistency is enforced. Select the variables u and v in each iteration of the while loop such that the domain of u changes in the call to `revise`$(\mathcal{C}, u, v)$. Provide u, v, and $\mathrm{dom}(u)$ in each iteration. Note that you do *not* have to provide the elements that are inserted into the queue, and you may stop the algorithm as soon as there are no variables u and v such that $\mathrm{dom}(u)$ changes.

Exercise 7.3 (1+3 marks)

Consider the constraint network $\mathcal{C} = \langle V, \mathrm{dom}, (R_{uv}) \rangle$ with $V = \{A, B, C, D, E, F, G\}$, $\mathrm{dom}(v) = \{1, \ldots, 10\}$ for all $v \in V$ and $R_{uv} = \mathrm{dom}(u) \times \mathrm{dom}(v)$ for all pairs $u, v \in V$ except for:

$$\begin{aligned}
R_{AC} &= \{\langle x, y \rangle \in \mathrm{dom}(A) \times \mathrm{dom}(C) \mid x \leq y\} \\
R_{BG} &= \{\langle x, y \rangle \in \mathrm{dom}(B) \times \mathrm{dom}(G) \mid x < y\} \\
R_{CE} &= \{\langle x, y \rangle \in \mathrm{dom}(C) \times \mathrm{dom}(E) \mid 2x < y\} \\
R_{CG} &= \{\langle x, y \rangle \in \mathrm{dom}(C) \times \mathrm{dom}(G) \mid |x - y| > 3\} \\
R_{DF} &= \{\langle x, y \rangle \in \mathrm{dom}(D) \times \mathrm{dom}(F) \mid |y - x^2| \leq 1\} \\
R_{FG} &= \{\langle x, y \rangle \in \mathrm{dom}(F) \times \mathrm{dom}(G) \mid |x - y| \leq 2\}
\end{aligned}$$

(a) Determine the constraint graph of $\mathcal{C}$.

(b) Apply the algorithm for trees as constraint graphs (slide 12 of chapter 27 of the print version of the lecture slides) to solve $\mathcal{C}$. Provide the following:

- the ordered tree, rooted at A;
- the variable ordering that is given by the ordered tree;
- the sequence of calls of the `revise` method and the resulting variable domains; and
- the result of a backtracking search with a value ordering that prefers smaller values. It suffices to provide the final variable assignments (it is, in particular, not necessary to provide the complete search tree).

Important: The exercise sheets can be submitted in groups of two students. Please provide both student names on the submission.