Foundations of Artificial Intelligence

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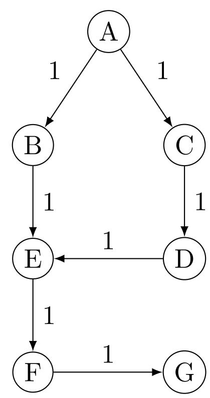
Exercise Sheet 6

Due: April 12, 2017

Exercise 6.1 (1.5+1 marks)

(a) $A^\star$ *without* reopening used with an admissible but *inconsistent* heuristic can find suboptimal solutions. Show this claim using a concrete example: specify a search problem (best as a graph) and a heuristic with the required properties for this problem. Then use $A^\star$ without reopening to solve the problem and to show that the solution found is not optimal. Specify the expansion order of the nodes with their g-, h-, and f-values.

Hint: You may use the following directed graph where the goal is to find a shortest path from A to G.



(b) Which part of the proof of optimality of $A^\star$ without reopening (chapter 19) becomes invalid if using an inconsistent heuristic? Justify your answer.

Exercise 6.2 (1.5+1.5 marks)

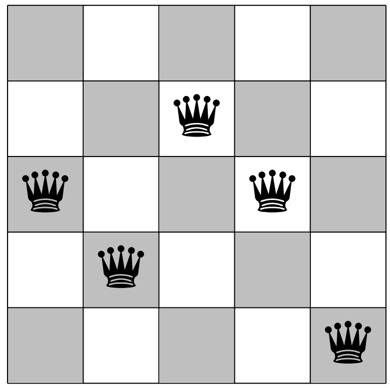
(a) A *vertex cover* of a given input graph G is a subset of the vertices of G such that every edge of G has at least one of its end points in the subset. Formalize the combinatorial optimization problem (COP) of finding a vertex cover of minimal size. Is the COP a pure search problem, a pure optimization problem, or a combined search and optimization problem?

(b) A *Hamilton path* of a directed graph G is a path through G that visits each vertex exactly once. The *longest Hamilton path* of a directed graph G with (positive) weighted edges is the Hamilton path with the maximal sum of edge weights.

Formalize the combinatorial optimization problem (COP) of finding a longest simple path in a graph that is **fully connected**, i.e., there is an edge between all pairs of vertices. Is the COP a pure search problem, a pure optimization problem, or a combined search and optimization problem?

Exercise 6.3 (2+2 marks)

The 5-queens problem is the COP on a 5×5 chess board that is analogous to the 8-queens problem that has been introduced in the lecture. In this exercise, we are interested in finding a solution for the 5-queens problem with different local search variants. Consider the following initial candidate:



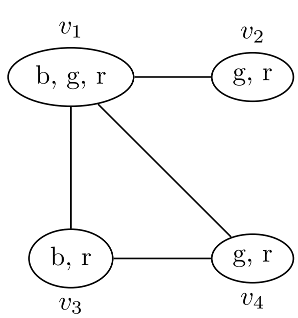
(a) Depict the candidates that are considered by hill climbing with the heuristic and the neighbourhood that are introduced on slide 22 of Chapter 20 in the print version of the lecture slides. If necessary, break ties in favor of candidates where the leftmost possible queen is moved to a square that is as far up as possible. Annotate the candidates that are considered during search with the heuristic values of all neighbours.

(b) Now consider a hill climbing variant where the next candidate that is considered is selected by first picking a file at random and then moving the (unique) queen in that file to the square in that file with the best heuristic value (the variant is introduced on slide 9 of Chapter 21 in the print version of the lecture slides). Assume that the random generator is such that the file selection picks the third file first, and in each subsequent step the file that is to the left of the file that had been selected last (if the leftmost file was selected last, the rightmost file is selected). Provide the candidates that are considered if the heuristic is used that counts the number of queens that threaten the queen in the selected file. If necessary, break ties in favor of candidates where the queen that is moved is as far up as possible. Annotate the candidates that are considered during search with the heuristic values of all neighbours.

Note that the file chess-board.tex, which you can find on the website, contains the LaTeX source code for the 5×5 chess board depicted above.

Exercise 6.4 (1+0.5+0.5+0.5 marks)

Consider the following example instance of the graph coloring problem:



(a) Formalize the example as a binary constraint network.

(b) Is the constraint network solvable? If yes, provide a solution of the constraint network. If not, justify your answer.

(c) Provide a consistent partial assignment that *cannot* be extended to a solution.

(d) Provide an inconsistent partial assignment.

Important: The exercise sheets can be submitted in groups of two students. Please provide both student names on the submission.