Theory of Computer Science

E4. Some NP-Complete Problems, Part I

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May 30, 2016

Overview: Course

contents of this course:

- logic ✓
  - ▷ How can knowledge be represented?
    How can reasoning be automated?
- automata theory and formal languages ✓
  - ▷ What is a computation?
- computability theory ✓
  - ▷ What can be computed at all?
- complexity theory
  - ▷ What can be computed efficiently?

Overview: Complexity Theory

Complexity Theory

E1. Motivation and Introduction

E2. P, NP and Polynomial Reductions

E3. Cook-Levin Theorem

E4. Some NP-Complete Problems, Part I

E5. Some NP-Complete Problems, Part II

Further Reading (German)

Literature for this Chapter (German)

Theoretische Informatik – kurz gefasst
by Uwe Schöning (5th edition)

- Chapter 3.3

Further Reading (English)

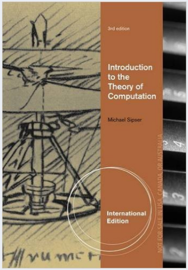
Literature for this Chapter (English)

Introduction to the Theory of Computation
by Michael Sipser (3rd edition)

- Chapter 7.4 and 7.5

Note:

- Sipser does not cover all problems that we do.



Overview

Further NP-Complete Problems

- The proof of NP-completeness for SAT was complicated.
- But with its help, we can now prove much more easily that further problems are NP-complete.

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- But with its help, we can now prove much more easily that further problems are NP-complete.

Theorem (Proving NP-Completeness by Reduction)

Let A and B be problems such that:

- *A is NP-hard, and*
- *$A \leq_p B$.*

Then B is also NP-hard.
If furthermore $B \in NP$, then B is NP-complete.

Proof.

First part shown in the exercises.
Second part follows directly by definition of NP-completeness. □

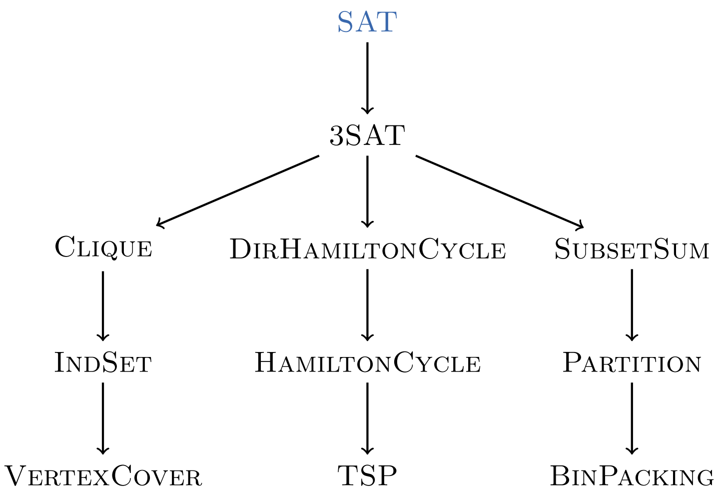
NP-Complete Problems

- There are thousands of known NP-complete problems.
- An extensive catalog of NP-complete problems from many areas of computer science is contained in:

  Michael R. Garey and David S. Johnson:
  Computers and Intractability —
  A Guide to the Theory of NP-Completeness
  W. H. Freeman, 1979.

- In the remaining two chapters, we get to know some of these problems.

Overview of the Reductions

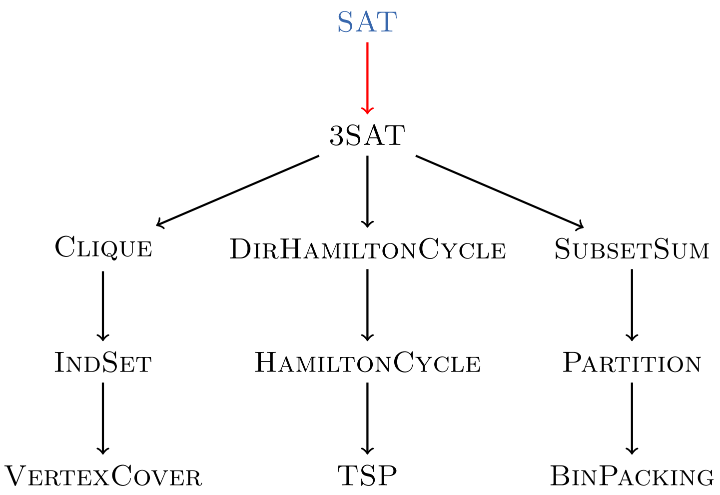


What Do We Have to Do?

- We want to show the NP-completeness of these 11 problems.
- We already know that SAT is NP-complete.
- Hence it is sufficient to show
  - that polynomial reductions exist for all edges in the figure (and thus all problems are NP-hard)
  - and that the problems are all in NP.

(It would be sufficient to show membership in NP only for the leaves in the figure. But membership is so easy to show that this would not save any work.)

SAT $\leq_p$ 3SAT



Questions

Questions?

3SAT

SAT and 3SAT

Definition (Reminder: SAT)

The problem SAT (satisfiability) is defined as follows:

Given: a propositional logic formula φ

Question: Is φ satisfiable?

Definition (3SAT)

The problem 3SAT is defined as follows:

Given: a propositional logic formula φ in conjunctive normal form with at most three literals per clause

Question: Is φ satisfiable?

3SAT is NP-Complete (1)

Theorem (3SAT is NP-Complete)

3SAT *is NP-complete.*

3SAT is NP-Complete (2)

Proof.

3SAT $\in$ NP: guess and check.

3SAT is NP-hard: We show SAT $\leq_p$ 3SAT.

- Let φ be the given input for SAT. Let $Sub(\varphi)$ denote the set of subformulas of φ, including φ itself.

3SAT is NP-Complete (2)

Proof.

$\mathrm{3SAT} \in \mathsf{NP}$: guess and check.

$\mathrm{3SAT}$ is NP-hard: We show $\mathrm{SAT} \leq_p \mathrm{3SAT}$.

- Let φ be the given input for SAT. Let $Sub(\varphi)$ denote the set of subformulas of φ, including φ itself.
- For all $\psi \in Sub(\varphi)$, we introduce a new proposition X_ψ.

$\mathrm{3SAT}$ is NP-Complete (2)

Proof.

$\mathrm{3SAT} \in$ NP: guess and check.

$\mathrm{3SAT}$ is NP-hard: We show $\mathrm{SAT} \leq_{\mathrm{p}} \mathrm{3SAT}$.

- Let φ be the given input for SAT. Let $\mathit{Sub}(\varphi)$ denote the set of subformulas of φ, including φ itself.
- For all $\psi \in \mathit{Sub}(\varphi)$, we introduce a new proposition X_ψ.
- For each new proposition X_ψ, define the following auxiliary formula χ_ψ:
  - If $\psi = A$ for an atom A: $\chi_\psi = (X_\psi \leftrightarrow A)$
  - If $\psi = \neg\psi'$: $\chi_\psi = (X_\psi \leftrightarrow \neg X_{\psi'})$
  - If $\psi = (\psi' \wedge \psi'')$: $\chi_\psi = (X_\psi \leftrightarrow (X_{\psi'} \wedge X_{\psi''}))$
  - If $\psi = (\psi' \vee \psi'')$: $\chi_\psi = (X_\psi \leftrightarrow (X_{\psi'} \vee X_{\psi''}))$

...

3SAT is NP-Complete (3)

Proof (continued).

- Consider the conjunction of all these auxiliary formulas, $\chi_{\mathsf{all}} := \bigwedge_{\psi \in \mathit{Sub}(\varphi)} \chi_\psi$.

3SAT is NP-Complete (3)

Proof (continued).

- Consider the conjunction of all these auxiliary formulas, $\chi_{\mathsf{all}} := \bigwedge_{\psi \in \mathit{Sub}(\varphi)} \chi_\psi$.
- Every interpretation $\mathcal{I}$ of the original variables can be extended to a model $\mathcal{I}'$ of χ_{all} in exactly one way: for each $\psi \in \mathit{Sub}(\varphi)$, set $\mathcal{I}'(X_\psi) = 1$ iff $\mathcal{I} \models \psi$.

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Proof (continued).

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- It follows that φ is satisfiable iff $(\chi_{\mathsf{all}} \land X_\varphi)$ is satisfiable.

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Proof (continued).

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- It follows that φ is satisfiable iff $(\chi_{\mathsf{all}} \wedge X_\varphi)$ is satisfiable.
- This formula can be computed in linear time.

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Proof (continued).

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- It follows that φ is satisfiable iff $(\chi_{\mathsf{all}} \land X_\varphi)$ is satisfiable.
- This formula can be computed in linear time.
- It can also be converted to 3-CNF in linear time because it is the conjunction of constant-size parts involving at most three variables each. (Each part can be converted to 3-CNF independently.)

3SAT is NP-Complete (3)

Proof (continued).

- Consider the conjunction of all these auxiliary formulas, $\chi_{\text{all}} := \bigwedge_{\psi \in \mathit{Sub}(\varphi)} \chi_\psi$.
- Every interpretation $\mathcal{I}$ of the original variables can be extended to a model $\mathcal{I}'$ of χ_{all} in exactly one way: for each $\psi \in \mathit{Sub}(\varphi)$, set $\mathcal{I}'(X_\psi) = 1$ iff $\mathcal{I} \models \psi$.
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- This formula can be computed in linear time.
- It can also be converted to 3-CNF in linear time because it is the conjunction of constant-size parts involving at most three variables each. (Each part can be converted to 3-CNF independently.)
- Hence, this describes a polynomial-time reduction.

□

Restricted 3SAT

Note: 3SAT remains NP-complete if we also require that

- every clause contains exactly three literals and
- a clause may not contain the same literal twice

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Idea:

- remove duplicated literals from each clause.
- add new variables: X, Y, Z
- add new clauses: $(X \lor Y \lor Z)$, $(X \lor Y \lor \neg Z)$, $(X \lor \neg Y \lor Z)$, $(\neg X \lor Y \lor Z)$, $(X \lor \neg Y \lor \neg Z)$, $(\neg X \lor Y \lor \neg Z)$, $(\neg X \lor \neg Y \lor Z)$

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$\rightsquigarrow$ satisfied if and only if X, Y, Z are all true

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Idea:

- remove duplicated literals from each clause.
- add new variables: X, Y, Z
- add new clauses: $(X \vee Y \vee Z)$, $(X \vee Y \vee \neg Z)$, $(X \vee \neg Y \vee Z)$, $(\neg X \vee Y \vee Z)$, $(X \vee \neg Y \vee \neg Z)$, $(\neg X \vee Y \vee \neg Z)$, $(\neg X \vee \neg Y \vee Z)$

$\rightsquigarrow$ satisfied if and only if X, Y, Z are all true

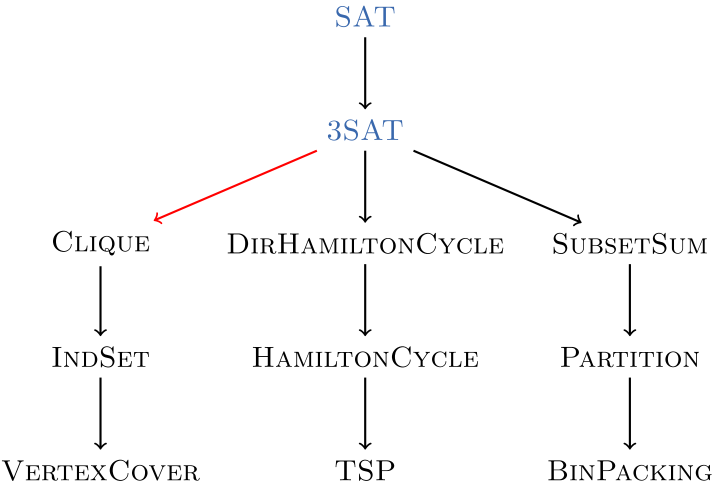
- fill up clauses with fewer than three literals with $\neg X$ and if necessary additionally with $\neg Y$

Questions

Questions?

Graph Problems

3SAT $\leq_p$ Clique



Clique

Definition (Clique)

The problem Clique is defined as follows:

Given: undirected graph $G = \langle V, E \rangle$, number $K \in \mathbb{N}_0$

Question: Does G have a clique of size at least K,
i. e., a set of vertices $C \subseteq V$ with $|C| \geq K$
and $\{u, v\} \in E$ for all $u, v \in C$ with $u \neq v$?

German: Clique

Clique is NP-Complete (1)

Theorem (Clique is NP-Complete)

Clique *is NP-complete.*

CLIQUE is NP-Complete (2)

Proof.

CLIQUE ∈ NP: guess and check.

CLIQUE is NP-Complete (2)

Proof.

CLIQUE $\in$ NP: guess and check.

CLIQUE is NP-hard: We show $3\text{SAT} \leq_p$ CLIQUE.

CLIQUE is NP-Complete (2)

Proof.

CLIQUE $\in$ NP: guess and check.

CLIQUE is NP-hard: We show $3\mathrm{SAT} \leq_{\mathrm{p}}$ CLIQUE.

- We are given a 3-CNF formula φ, and we may assume that each clause has exactly three literals.
- In polynomial time, we must construct a graph $G = \langle V, E \rangle$ and a number K such that: G has a clique of size at least K iff φ is satisfiable.

CLIQUE is NP-Complete (2)

Proof.

CLIQUE $\in$ NP: guess and check.

CLIQUE is NP-hard: We show $\mathrm{3SAT} \leq_{\mathrm{p}}$ CLIQUE.

- We are given a 3-CNF formula φ, and we may assume that each clause has exactly three literals.
- In polynomial time, we must construct a graph $G = \langle V, E \rangle$ and a number K such that: G has a clique of size at least K iff φ is satisfiable.

$\rightsquigarrow$ construction of V, E, K on the following slides.

...

CLIQUE is NP-Complete (3)

Proof (continued).

Let m be the number of clauses in φ.

Let l_{ij} the j-th literal in clause i.

CLIQUE is NP-Complete (3)

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Define V, E, K as follows:

CLIQUE is NP-Complete (3)

Proof (continued).

Let m be the number of clauses in φ.

Let l_{ij} the j-th literal in clause i.

Define V, E, K as follows:

- $V = \{\langle i, j\rangle \mid 1 \leq i \leq m, 1 \leq j \leq 3\}$
  $\rightsquigarrow$ a vertex for every literal of every clause

CLIQUE is NP-Complete (3)

Proof (continued).

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Define V, E, K as follows:

- $V = \{\langle i, j \rangle \mid 1 \leq i \leq m, 1 \leq j \leq 3\}$
  $\rightsquigarrow$ a vertex for every literal of every clause
- E contains edge between $\langle i, j \rangle$ and $\langle i', j' \rangle$ if and only if
  - $i \neq i'$ $\rightsquigarrow$ belong to different clauses, and
  - l_{ij} and $l_{i'j'}$ are not complementary literals

CLIQUE is NP-Complete (3)

Proof (continued).

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$\rightsquigarrow$ obviously polynomially computable

CLIQUE is NP-Complete (3)

Proof (continued).

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- $K = m$

$\rightsquigarrow$ obviously polynomially computable

to show: reduction property . . .

CLIQUE is NP-Complete (4)

Proof (continued).

(⇒): If φ is satisfiable, then $\langle V, E\rangle$ has clique of size at least K:

CLIQUE is NP-Complete (4)

Proof (continued).

($\Rightarrow$): If φ is satisfiable, then $\langle V, E \rangle$ has clique of size at least K:

- Given a satisfying variable assignment choose a vertex corresponding to a satisfied literal in each clause.
- The chosen K vertices are all connected with each other and hence form a clique of size K.

...

CLIQUE is NP-Complete (5)

Proof (continued).

($\Leftarrow$): If $\langle V, E \rangle$ has a clique of size at least K, then φ is satisfiable:

CLIQUE is NP-Complete (5)

Proof (continued).

(⇐): If $\langle V, E \rangle$ has a clique of size at least K, then φ is satisfiable:

- Consider a given clique C of size at least K.
- The vertices in C must all correspond to different clauses (vertices in the same clause are not connected by edges).

$\rightsquigarrow$ exactly one vertex per clause is included in C

- Two vertices in C never correspond to complementary literals X and $\neg X$ (due to the way we defined the edges).

CLIQUE is NP-Complete (5)

Proof (continued).

($\Leftarrow$): If $\langle V, E \rangle$ has a clique of size at least K, then φ is satisfiable:

- Consider a given clique C of size at least K.
- The vertices in C must all correspond to different clauses (vertices in the same clause are not connected by edges).

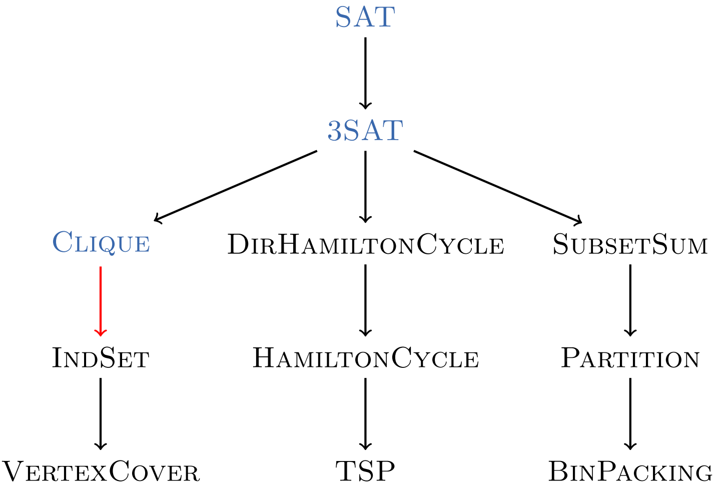
$\rightsquigarrow$ exactly one vertex per clause is included in C

- Two vertices in C never correspond to complementary literals X and $\neg X$ (due to the way we defined the edges).
- If a vertex corresp. to X was chosen, map X to 1 (true).
- If a vertex corresp. to $\neg X$ was chosen, map X to 0 (false).
- If neither was chosen, arbitrarily map X to 0 or 1.

$\rightsquigarrow$ satisfying assignment

$\square$

CLIQUE $\leq_p$ INDSET



IndSet

Definition (IndSet)

The problem IndSet is defined as follows:

Given: undirected graph $G = \langle V, E \rangle$, number $K \in \mathbb{N}_0$

Question: Does G have an independent set of size at least K, i. e., a set of vertices $I \subseteq V$ with $|I| \geq K$ and $\{u, v\} \notin E$ for all $u, v \in I$ with $u \neq v$?

German: unabhängige Menge

IndSet is NP-Complete (1)

Theorem (IndSet is NP-Complete)

IndSet *is NP-complete.*

IndSet is NP-Complete (2)

Proof.

IndSet $\in$ NP: guess and check.

INDSET is NP-Complete (2)

Proof.

INDSET $\in$ NP: guess and check.

INDSET is NP-hard: We show CLIQUE $\leq_p$ INDSET.

IndSet is NP-Complete (2)

Proof.

IndSet $\in$ NP: guess and check.

IndSet is NP-hard: We show Clique $\leq_p$ IndSet.

We describe a polynomial reduction f.
Let $\langle G, K \rangle$ with $G = \langle V, E \rangle$ be the given input for Clique.

IndSet is NP-Complete (2)

Proof.

IndSet $\in$ NP: guess and check.

IndSet is NP-hard: We show Clique $\leq_p$ IndSet.

We describe a polynomial reduction f.
Let $\langle G, K\rangle$ with $G = \langle V, E\rangle$ be the given input for Clique.

Then $f(\langle G, K\rangle)$ is the IndSet instance $\langle \overline{G}, K\rangle$, where
$\overline{G} := \langle V, \overline{E}\rangle$ and $\overline{E} := \{\{u, v\} \subseteq V \mid u \neq v, \{u, v\} \notin E\}$.
(This graph $\overline{G}$ is called the complement graph of G.)

IndSet is NP-Complete (2)

Proof.

IndSet $\in$ NP: guess and check.

IndSet is NP-hard: We show Clique $\leq_p$ IndSet.

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Let $\langle G, K \rangle$ with $G = \langle V, E \rangle$ be the given input for Clique.

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(This graph $\overline{G}$ is called the complement graph of G.)

Clearly f can be computed in polynomial time. ...

IndSet is NP-Complete (3)

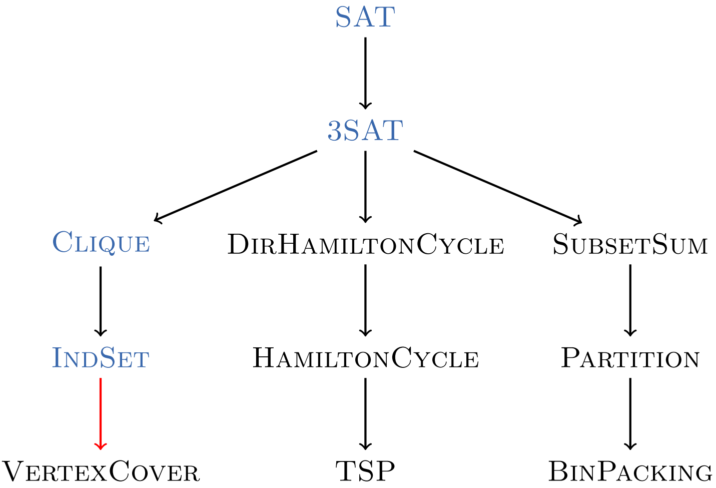
Proof (continued).

We have:

$$
\begin{aligned}
& \langle\langle V, E\rangle, K\rangle \in \textsc{Clique} \\
\text{iff}\quad & \text{there exists a set } V' \subseteq V \text{ with } |V'| \geq K \\
& \quad \text{and } \{u, v\} \in E \text{ for all } u, v \in V' \text{ with } u \neq v \\
\text{iff}\quad & \text{there exists a set } V' \subseteq V \text{ with } |V'| \geq K \\
& \quad \text{and } \{u, v\} \notin \overline{E} \text{ for all } u, v \in V' \text{ with } u \neq v \\
\text{iff}\quad & \langle\langle V, \overline{E}\rangle, K\rangle \in \textsc{IndSet} \\
\text{iff}\quad & f(\langle\langle V, E\rangle, K\rangle) \in \textsc{IndSet}
\end{aligned}
$$

and hence f is a reduction. □

IndSet $\leq_p$ VertexCover



VertexCover

Definition (VertexCover)

The problem VertexCover is defined as follows:

Given: undirected graph $G = \langle V, E \rangle$, number $K \in \mathbb{N}_0$

Question: Does G have a vertex cover of size at most K, i. e., a set of vertices $C \subseteq V$ with $|C| \leq K$ and $\{u, v\} \cap C \neq \emptyset$ for all $\{u, v\} \in E$?

German: Knotenüberdeckung

VertexCover is NP-Complete (1)

Theorem (VertexCover is NP-Complete)

VertexCover *is NP-complete.*

VertexCover is NP-Complete (2)

Proof.

VertexCover $\in$ NP: guess and check.

VertexCover is NP-Complete (2)

Proof.

VertexCover $\in$ NP: guess and check.

VertexCover is NP-hard:
We show IndSet $\leq_p$ VertexCover.

VertexCover is NP-Complete (2)

Proof.

VertexCover $\in$ NP: guess and check.

VertexCover is NP-hard:
We show IndSet $\leq_p$ VertexCover.

We describe a polynomial reduction f.
Let $\langle G, K \rangle$ with $G = \langle V, E \rangle$ be the given input for IndSet.

VertexCover is NP-Complete (2)

Proof.

VertexCover $\in$ NP: guess and check.

VertexCover is NP-hard:
We show IndSet $\leq_p$ VertexCover.

We describe a polynomial reduction f.
Let $\langle G, K\rangle$ with $G = \langle V, E\rangle$ be the given input for IndSet.

Then $f(\langle G, K\rangle) := \langle G, |V| - K\rangle$.
This can clearly be computed in polynomial time. . . .

VertexCover is NP-Complete (3)

Proof (continued).

For vertex set $V' \subseteq V$, we write $\overline{V'}$ for its complement $V \setminus V'$.

VertexCover is NP-Complete (3)

Proof (continued).

For vertex set $V' \subseteq V$, we write $\overline{V'}$ for its complement $V \setminus V'$.

Observation: a set of vertices is a vertex cover
iff its complement is an independent set.

$\textsc{VertexCover}$ is NP-Complete (3)

Proof (continued).

For vertex set $V' \subseteq V$, we write $\overline{V'}$ for its complement $V \setminus V'$.

Observation: a set of vertices is a vertex cover iff its complement is an independent set.

We thus have:

$$
\begin{aligned}
& \langle\langle V, E\rangle, K\rangle \in \textsc{IndSet} \\
\text{iff} \quad & \langle V, E\rangle \text{ has an independent set } I \text{ with } |I| \geq K \\
\text{iff} \quad & \langle V, E\rangle \text{ has a vertex cover } C \text{ with } |\overline{C}| \geq K \\
\text{iff} \quad & \langle V, E\rangle \text{ has a vertex cover } C \text{ with } |C| \leq |V| - K \\
\text{iff} \quad & \langle\langle V, E\rangle, |V| - K\rangle \in \textsc{VertexCover} \\
\text{iff} \quad & f(\langle\langle V, E\rangle, K\rangle) \in \textsc{VertexCover}
\end{aligned}
$$

and hence f is a reduction. □

Questions

Questions?

Summary

Summary

- Thousands of important problems are NP-complete.
- Usually, the easiest way to show that a problem is NP-complete is to
  - show that it is in NP with a guess-and-check algorithm, and
  - polynomially reduce a known NP-complete to it.
- In this chapter we showed NP-completeness of:
  - 3SAT, a restricted version of SAT
  - three classical graph problems: CLIQUE, INDSET, VERTEXCOVER