Theory of Computer Science

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Exercise Sheet 14

Due: Wednesday, June 8, 2016

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Note: All exercises on this exercise sheet are bonus exercises.

Exercise 14.1 (NP-Completeness, 1.5 Bonus Points)

The following statements are all wrong. In each case, explain in 1–2 sentences why the statement is wrong and what a correct version would be.

(a) To show that a problem X is NP-complete, it suffices to show that $X \in \mathrm{NP}$ and $X \leq_{\mathrm{p}} Y$ for some NP-complete problem Y.

(b) There is a NP-complete problem X that can be solved with an efficient deterministic algorithm, even if there is none for SAT.

(c) For every NP-hard problem X: $X \leq_{\mathrm{p}}$ SAT.

Exercise 14.2 (PARTITION $\leq_{\mathrm{p}}$ SUBSETSUM, 1.5 Bonus Points)

Consider the decision problems PARTITION and SUBSETSUM from chapter E5.
Reduce PARTITION to SUBSETSUM with a polynomial reduction to show PARTITION $\leq_{\mathrm{p}}$ SUBSETSUM.

Exercise 14.3 (HITTINGSET, 1+2 Bonus Points)

Consider the decision problem HITTINGSET:

- *Given:* A finite set T, a set of sets $S = \{S_1, \dots, S_n\}$ with $S_i \subseteq T$ for all $i \in \{1, \dots, n\}$, a natural number $K \in \mathbb{N}_0$ with $K \leq |T|$.
- *Question:* Is there a set H with at most K elements that contains at least one element from each set in S?

(a) Prove that HITTINGSET is in NP by specifying a non-deterministic algorithm for HITTINGSET whose runtime is limited by a polynomial in $n|T|$.

(b) Prove that HITTINGSET is NP-complete. You may use without proof that the problem VERTEXCOVER (from chapter E5) is NP-complete.

Exercise 14.4 (Variants of DirHamiltonPath, 2+2 Bonus Points)

Consider the decision problems DirHamiltonPath, DirHamiltonPathFromVertex and DirHamiltonPathToVertex:

DirHamiltonPath (see exercise 13.3):

- *Given:* directed graph $G = \langle V, E\rangle$
- *Question:* Does G contain a Hamilton path?

DirHamiltonPathFromVertex

- *Given:* directed graph $G = \langle V, E\rangle$, vertex $v_{\text{start}} \in V$
- *Question:* Does G contain a Hamilton path that starts in v_{start}?

DirHamiltonPathToVertex

- *Given:* directed graph $G = \langle V, E\rangle$, vertex $v_{\text{end}} \in V$
- *Question:* Does G contain a Hamilton path that ends in v_{end}?

(a) Reduce DirHamiltonPath to DirHamiltonPathFromVertex with a polynomial reduction to show

$$\textsc{DirHamiltonPath} \leq_{\mathrm{p}} \textsc{DirHamiltonPathFromVertex}.$$

(b) Reduce DirHamiltonPathFromVertex to DirHamiltonPathToVertex with a polynomial reduction to show

$$\textsc{DirHamiltonPathFromVertex} \leq_{\mathrm{p}} \textsc{DirHamiltonPathToVertex}.$$