Theory of Computer Science

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Exercise Sheet 12

Due: Wednesday, May 25, 2016

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Exercise 12.1 (Emptiness Problem, 2 Points)

The *emptiness problem* for general grammars (i.e. type-0 grammars) is defined as:

EMPTINESS: Given a general grammar G, is $\mathcal{L}(G) = \emptyset$?

Prove that EMPTINESS is undecidable.

Hints: you can use without proof that there is a computable function that transforms a given type-0 grammar G to a DTM M_G with $\mathcal{L}(M_G) = \mathcal{L}(G)$. Likewise, there is a computable function that transforms a given DTM M to a type-0 grammar G_M with $\mathcal{L}(M) = \mathcal{L}(G_M)$.
Use Rice's theorem in an appropriate way to show the undecidability.

Exercise 12.2 (Undecidable Grammar Problems, 1.5+1.5 Points)

The *equivalence problem* and the *intersection problem* for general grammars are defined as:

- EQUIVALENCE: Given two general grammars G_1 and G_2, is $\mathcal{L}(G_1) = \mathcal{L}(G_2)$?
- INTERSECTION: Given two general grammars G_1 and G_2, is $\mathcal{L}(G_1) \cap \mathcal{L}(G_2) = \emptyset$?

(a) Show that EQUIVALENCE is undecidable, by reducing EMPTINESS to it.

(b) Show that INTERSECTION is undecidable, by reducing EMPTINESS to it.

Hint: of course you can use the fact that EMPTINESS is undecidable even if you did not complete exercise 12.1.

Exercise 12.3 (Rice's Theorem, 1 Bonus Point)

For which of the following languages does Rice's theorem show that the language is undecidable? For each language where Rice's theorem can be used, specify the subset of Turing-computable functions $\mathcal{S}$ for which you use the theorem.
Hint: You do not have to write down any proofs. If Rice's theorem is applicable, specify the set $\mathcal{S}$, otherwise give a short reason (1 sentence) why Rice's theorem is not applicable.

(a) $L = \{w \in \{0,1\}^* \mid M_w \text{ computes the binary multiplication function}\}$

(b) $L = \{w \in \{0,1\}^* \mid \text{The output of } M_w \text{ started on the empty tape contains } \texttt{0101}\}$

(c) $L = \{w \in \{0,1\}^* \mid M_w \text{ stops for at least one input after more than 10 steps with a valid output}\}$

(d) $L = \{w \in \{0,1\}^* \mid M_w \text{ computes a binary function over the natural numbers}\}$