**Theory of Computer Science**

M. Helmert
F. Pommerening
Spring Term 2016

University of Basel
Computer Science

# Exercise Sheet 10

**Due: Wednesday, May 11, 2016**

*Note:* Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

**Exercise 10.1** (Power Function; 1 Point)

Prove that the following definition of the power function is correct, that is that $pow(x, y) = x^y$ for all $x, y \in \mathbb{N}_0$ (we define the special case $0^0$ as $0^0 = 1$ for this exercise). Also specify definitions of $h$ and $pow_{rev}$ in common mathematical notation.

$$\begin{aligned} h &= compose(mul, \pi_1^3, \pi_3^3) \\ pow_{rev} &= primitive\_recursion(one, h) \\ pow &= compose(pow_{rev}, \pi_2^2, \pi_1^2) \end{aligned}$$

The function $mul : \mathbb{N}_0^2 \to \mathbb{N}_0$ is the regular multiplication function: $mul(x, y) = x \cdot y$ for all $x, y \in \mathbb{N}_0$ and the function $one : \mathbb{N}_0 \to \mathbb{N}_0$ the constant one function: $one(x) = 1$ for all $x \in \mathbb{N}_0$.

**Exercise 10.2** (Primitive-Recursive Functions; 4 Points)

On the course website you will find a Java program where you can define and evaluate primitive-recursive functions. Define the following functions with the help of this program, but without using the $\mu$-operator (thus showing that the functions are primitive-recursive). In each case, demonstrate the correctness of your definition for some examples using the `print` function.

*Note:* In your solution you may use all functions which were defined in the lecture. The file `lecture.def` contains their definitions. Add your definitions at the end of this file below the corresponding marker. Any changes above the marker will be ignored.

(a) $add\_succ : \mathbb{N}_0^2 \to \mathbb{N}_0$ with $add\_succ(x, y) = x + y + 1$ for all $x, y \in \mathbb{N}_0$.

(b) $binom_2 : \mathbb{N}_0 \to \mathbb{N}_0$ with $binom_2(x) = \binom{x}{2}$ for all $x \in \mathbb{N}_0$

You can use without proof that for all $n \in \mathbb{N}_0$: $\sum_{i=1}^{n} i = \binom{n+1}{2}$

(c) $encode : \mathbb{N}_0^2 \to \mathbb{N}_0$ with $encode(x, y) = \binom{x+y+1}{2} + x$ for all $x, y \in \mathbb{N}_0$.

(d) $fac : \mathbb{N}_0 \to \mathbb{N}_0$ with $fac(x) = x!$ for all $x \in \mathbb{N}_0$.

**Exercise 10.3** ($\mu$-Operator; 3 Points)

For each of the following functions $f$ specify a definition of $\mu f$ in common mathematical notation.

(a) $f(x, y) = y \ominus x^3$ for all $x, y \in \mathbb{N}_0$

(b) $f(x, y) = (y^2 \ominus x) \cdot (10 \ominus x)$ for all $x, y \in \mathbb{N}_0$

(c) $f(x, y) = |x - 2^y + 5|$ for all $x, y \in \mathbb{N}_0$

**Exercise 10.4** ($\mu$-rekursive Functions; 2 Points)

Show with the program from exercise 10.2 that the function

$$\max : \mathbb{N}_0^2 \to \mathbb{N}_0 \text{ with } \max(x, y) = \begin{cases} x & \text{if } x \geq y \\ y & \text{otherwise} \end{cases} \quad \text{for all } x, y \in \mathbb{N}_0$$

is $\mu$-recursive. You may use all $\mu$-recursive functions shown in the lecture. Demonstrate the correctness of your definition for some examples using the `print` function.