Theory of Computer Science

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Exercise Sheet 7

Due: Wednesday, April 20, 2016

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Exercise 7.1 (ε-Rules; 2 Points)

Consider the grammar $G = \langle \Sigma, V, P, \mathrm{S} \rangle$ with $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, $V = \{\mathrm{S}, \mathrm{X}, \mathrm{Y}, \mathrm{Z}\}$ and the following rules in P:

$$\mathrm{S} \to \mathrm{X} \qquad \mathrm{X} \to \mathrm{Y} \qquad \mathrm{X} \to \mathrm{XZY} \qquad \mathrm{Y} \to \mathtt{aXbYcS} \qquad \mathrm{Y} \to \varepsilon \qquad \mathrm{Z} \to \mathrm{Z}\mathtt{bb} \qquad \mathrm{Z} \to \mathtt{bb}$$

Specify $V_\varepsilon = \{A \in V \mid A \Rightarrow^* \varepsilon\}$ and construct a grammar G' with $\mathcal{L}(G') = \mathcal{L}(G)$ according to the rules on slides 5–7 of chapter C5. In the rules of G', ε may only occur in the rule $\mathrm{S} \to \varepsilon$ if S is the start symbol and never occurs on the right-hand side of a rule.
Specify sufficient intermediate steps, so your construction is understandable.

Exercise 7.2 (Chomsky Normal Form; 2 Points)

Specify a grammar G' in Chomsky normal form that generates the same language as the context-free grammar $G = \langle \Sigma, V, P, \mathrm{S} \rangle$ with $\Sigma = \{\mathtt{a}, \mathtt{b}\}$, $V = \{\mathrm{S}, \mathrm{X}, \mathrm{Y}, \mathrm{Z}\}$ and the following rules in P:

$$\mathrm{S} \to \varepsilon \qquad \mathrm{S} \to \mathrm{XY} \qquad \mathrm{S} \to \mathrm{Z} \qquad \mathrm{X} \to \mathrm{Y} \qquad \mathrm{X} \to \mathtt{aZb}$$
$$\mathrm{Y} \to \mathrm{X} \qquad \mathrm{Y} \to \mathtt{b}\mathrm{Y} \qquad \mathrm{Z} \to \mathtt{bb} \qquad \mathrm{Z} \to \mathrm{Z}\mathtt{a}$$

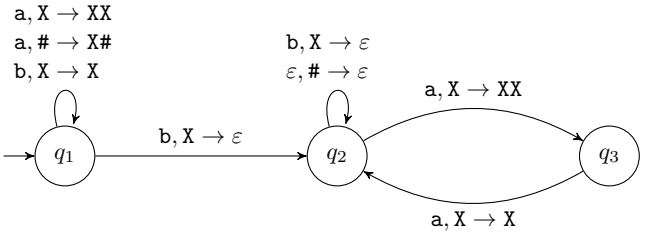
Specify sufficient intermediate steps, so your construction is understandable.

Exercise 7.3 (Length of Derivations in Chomsky Normal Form; 2 Points)

Let G be a grammar in Chomsky normal form and $w \in \mathcal{L}(G)$ a non-empty word ($w \neq \varepsilon$), which is generated by G. Show that every derivation of w from the start variable of G consists of exactly $2|w| - 1$ steps.

Exercise 7.4 (PDAs; 1+3 Points)

(a) Consider the PDA $M = \langle \{q_1, q_2, q_3\}, \{\mathtt{a}, \mathtt{b}\}, \{\mathtt{X}, \mathtt{\#}\}, \delta, q_1, \mathtt{\#} \rangle$ with the following transition function δ:



Prove that M accepts the word aababbaabb by specifying a sequence of configurations as defined on slides 12 and 13 of chapter C6.

(b) Specify a PDA that accepts the language $L = \{(\mathtt{ab})^n \mathtt{c} \mathtt{a}^n \mathtt{c} \mid n \geq 0\}$ over $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$.