Theory of Computer Science

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Exercise Sheet 4

Due: Wednesday, March 30, 2016

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Exercise 4.1 (Free and Bound Variables; 1 Point)

Consider the formula φ over a signature with predicate symbols P (1-ary), Q (2-ary) and R (3-ary), the 1-ary function symbol f, the constant symbol c and the variable symbols x, y and z.

$$\varphi = (\mathrm{P}(z) \rightarrow (\forall x\, \mathrm{P}(x) \wedge \exists y\, ((\mathrm{Q}(x, y) \vee \forall y \mathrm{R}(c, \mathrm{f}(z), y)) \vee \mathrm{P}(y))))$$

Mark all occurrences of bound variables in φ and specify the quantifier that binds the variable in each case.
Additionally specify the set of free variables of φ (without proof).

Exercise 4.2 (Logical Consequence in Predicate Logic; 1+1+1 Points)

Prove the following statements for predicate logic formulas φ and ψ.

(a) $(\forall x\, \varphi \wedge \forall x\, \psi) \equiv \forall x\, (\varphi \wedge \psi)$

(b) $\exists x\, (\varphi \wedge \psi) \models (\exists x\, \varphi \wedge \exists x\, \psi)$

(c) $(\exists x\, \varphi \wedge \exists x\, \psi) \not\models \exists x\, (\varphi \wedge \psi)$

Exercise 4.3 (Properties of Formulas in Predicate Logic; 1 Point + 1 Bonus Point)

Consider the following set of predicate logic formulas over a signature with the binary predicate symbol P and the variable symbols x and y.

$$\Phi = \{\forall x \exists y\, \mathrm{P}(x, y), \exists y \forall x\, \neg \mathrm{P}(x, y)\}$$

Which of the properties *satisfiable*, *unsatisfiable*, *valid*, and *falsifiable* are true for Φ? Jusitfy your answer for each of the four properties.

Bonus point: prove your answers.

Note: a formal proof is only required for the bonus point. If you don't want to do a formal proof, it is sufficient if you specify a universe U and an interpretation of P (that is $\mathrm{P}^{\mathcal{I}}$) in each case and briefly argue why $\mathcal{I} \models \Phi$ or $\mathcal{I} \not\models \Phi$ is true.