Theory of Computer Science

M. Helmert
F. Pommerening
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University of Basel
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Exercise Sheet 3

Due: Wednesday, March 16, 2015

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Exercise 3.1 (Equivalences; 1.5+1.5 Points)

(a) Transform the following formula into CNF by applying the equivalence rules shown in the lecture. For each step, only apply one equivalence rule and also specify it.

$$\varphi = ((\mathrm{A} \rightarrow \mathrm{B}) \leftrightarrow \neg \mathrm{C})$$

(b) Prove that the following formula is unsatisfiable by showing that $\varphi \equiv (\mathrm{A} \wedge \neg \mathrm{A})$ holds. Use the equivalence rules from the lectures, only apply one rule for each step and specify the applied rule.

$$\varphi = \neg((\mathrm{A} \wedge (\neg \mathrm{B} \rightarrow \mathrm{A})) \vee \neg \mathrm{A})$$

Exercise 3.2 (Logical Consequence; 1.5+1.5 Points)
Consider the following formula set over $\{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$.

$$\mathrm{KB} = \{(\mathrm{A} \rightarrow \neg \mathrm{C}), (\mathrm{A} \vee \neg \mathrm{B}), (\neg \mathrm{A} \vee \mathrm{C})\}$$

(a) Does a model $\mathcal{I}$ of KB exist which is also a model for $\varphi = (\mathrm{A} \vee \mathrm{B})$? Prove your statement.

(b) Prove that all models $\mathcal{I}$ of KB are also models of $\varphi = (\neg \mathrm{B} \vee \mathrm{C})$.

Exercise 3.3 ((Contraposition Theorem; 2 Points)
Prove the contraposition theorem, that is, show for any set of formulas KB and any formulas φ and ψ that

$$\mathrm{KB} \cup \{\varphi\} \models \neg \psi \text{ iff } \mathrm{KB} \cup \{\psi\} \models \neg \varphi.$$

Hint: you may use the deduction theorem.

Exercise 3.4 (Predicate Logic; 2 Points)
Consider the following predicate logic formula φ with the signature $\langle \{x\}, \{\mathrm{c}\}, \{\mathrm{f}\}, \{\mathrm{P}\} \rangle$.

$$\varphi = (\exists x \, (\mathrm{P}(x) \wedge \neg \mathrm{P}(\mathrm{f}(x))) \wedge \forall x \, \neg(\mathrm{f}(x) = \mathrm{c}))$$

Specify a model $\mathcal{I}$ of φ with $\mathcal{I} = \langle U, \cdot^{\mathcal{I}} \rangle$ and $U = \{u_1, u_2, u_3\}$. Prove that $\mathcal{I} \models \varphi$. Why is no variable assignment α required to specify a model of φ?