# Foundations of Artificial Intelligence

## 40. Automated Planning: Landmark Heuristics

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# Automated Planning: Overview

Chapter overview: planning

- 33. Introduction
- 34. Planning Formalisms
- 35.–36. Planning Heuristics: Delete Relaxation
- 37.–38. Planning Heuristics: Abstraction
- 39.–40. Planning Heuristics: Landmarks
  - 39. Landmarks
  - 40. Landmark Heuristics

# Formalism and Example

- As in the previous chapter, we consider delete-free planning tasks in normal form.
- We continue with the example from the previous chapter:

**Example**

actions:

- $a_1 = \langle i \to x, y \rangle_3$
- $a_2 = \langle i \to x, z \rangle_4$
- $a_3 = \langle i \to y, z \rangle_5$
- $a_4 = \langle x, y, z \to g \rangle_0$

landmark examples:

- $A = \{a_4\}$ (cost $= 0$)
- $B = \{a_1, a_2\}$ (cost $= 3$)
- $C = \{a_1, a_3\}$ (cost $= 3$)
- $D = \{a_2, a_3\}$ (cost $= 4$)

# Finding Landmarks

# Justification Graphs

## Definition (precondition choice function)

A precondition choice function (pcf) $P : A \to V$ maps every action to one of its preconditions.

## Definition (justification graph)

The justification graph for pcf $P$ is a directed graph with annotated edges.

- vertices: the variables $V$
- edges: $P(a) \xrightarrow{a} e$ for every action $a$, every effect $e \in add(a)$

# Example: Justification Graph

**Example**

pcf $P$: $P(a_1) = P(a_2) = P(a_3) = i$, $P(a_4) = y$

$a_1 = \langle i \to x, y \rangle_3$
$a_2 = \langle i \to x, z \rangle_4$
$a_3 = \langle i \to y, z \rangle_5$
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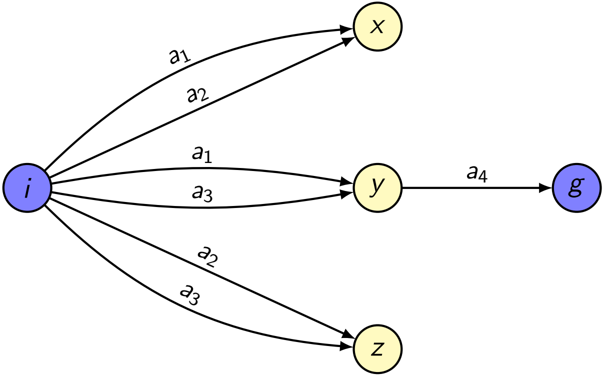

# Cuts

**Definition (cut)**

A cut in a justification graph is a subset $C$ of its edges such that all paths from $i$ to $g$ contain an edge in $C$.

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## Proposition (cuts are landmarks)

*Let $C$ be a cut in a justification graph for an arbitrary pcf.*

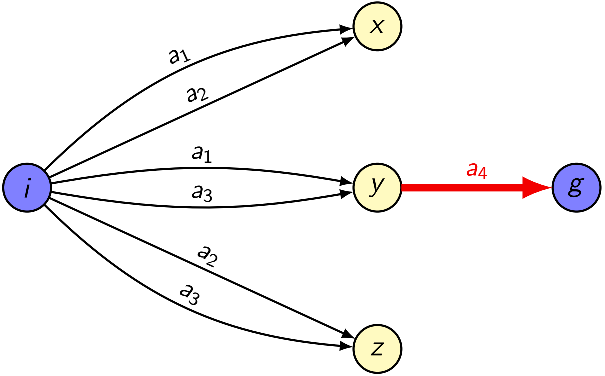
*Then the edge annotations for $C$ form a landmark.*

# Example: Cuts in Justification Graphs

## Example

landmark $A = \{a_4\}$ (cost $= 0$)

$a_1 = \langle i \rightarrow x, y \rangle_3$
$a_2 = \langle i \rightarrow x, z \rangle_4$
$a_3 = \langle i \rightarrow y, z \rangle_5$
$a_4 = \langle x, y, z \rightarrow g \rangle_0$

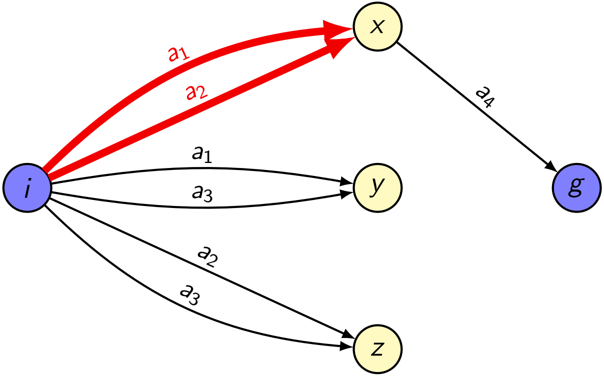


# Example: Cuts in Justification Graphs

## Example

landmark $B = \{a_1, a_2\}$ (cost $= 3$)

$a_1 = \langle i \rightarrow x, y \rangle_3$
$a_2 = \langle i \rightarrow x, z \rangle_4$
$a_3 = \langle i \rightarrow y, z \rangle_5$
$a_4 = \langle x, y, z \rightarrow g \rangle_0$

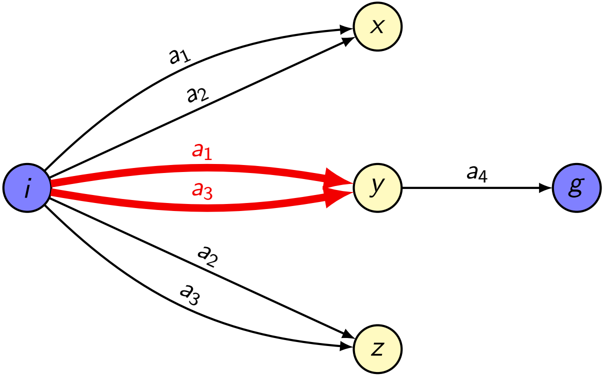


# Example: Cuts in Justification Graphs

**Example**

landmark $C = \{a_1, a_3\}$ (cost = 3)

$a_1 = \langle i \rightarrow x, y \rangle_3$
$a_2 = \langle i \rightarrow x, z \rangle_4$
$a_3 = \langle i \rightarrow y, z \rangle_5$
$a_4 = \langle x, y, z \rightarrow g \rangle_0$

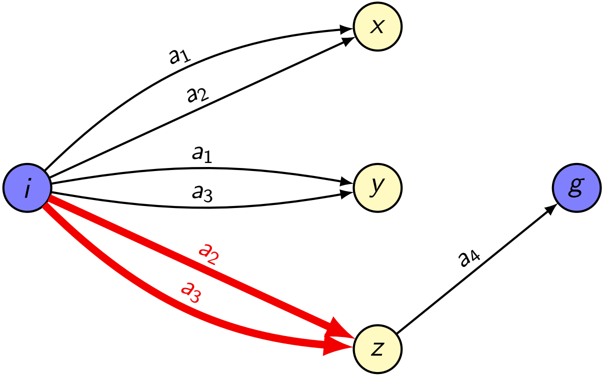


# Example: Cuts in Justification Graphs

**Example**

landmark $D = \{a_2, a_3\}$ (cost $= 4$)

$a_1 = \langle i \rightarrow x, y \rangle_3$
$a_2 = \langle i \rightarrow x, z \rangle_4$
$a_3 = \langle i \rightarrow y, z \rangle_5$
$a_4 = \langle x, y, z \rightarrow g \rangle_0$



# Power of Cuts in Justification Graphs

- Which landmarks can be computed with the cut method?

# Power of Cuts in Justification Graphs

- Which landmarks can be computed with the cut method?
- all interesting ones!

**Proposition (perfect hitting set heuristics)**

*Let $\mathcal{L}$ be the set of all "cut landmarks" of a given planning task. Then $h^{\mathrm{MHS}}(I) = h^{+}(I)$ for $\mathcal{L}$.*

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proof idea:

- Show 1:1 correspondence of hitting sets $H$ for $\mathcal{L}$ and plans, i.e., each hitting set for $\mathcal{L}$ corresponds to a plan, and vice versa.

# The LM-Cut Heuristic

# LM-Cut Heuristic: Motivation

- In general, there are exponentially many pcfs, hence computing all relevant landmarks is not tractable.
- The LM-cut heuristic is a method that chooses pcfs and computes cuts in a goal-oriented way.
- A cost partitioning is computed as a side effect and is usually not optimal.
- On the other hand, it can be computed efficiently and is optimal for planning tasks with uniform costs (i.e., $cost(a) = 1$ for all actions).

$\rightsquigarrow$ currently the best admissible planning heuristic

# The LM-Cut Heuristic

## $h^{\text{LM-cut}}$: Helmert & Domshlak (2009)

Initialize $h^{\text{LM-cut}}(I) := 0$. Then iterate:

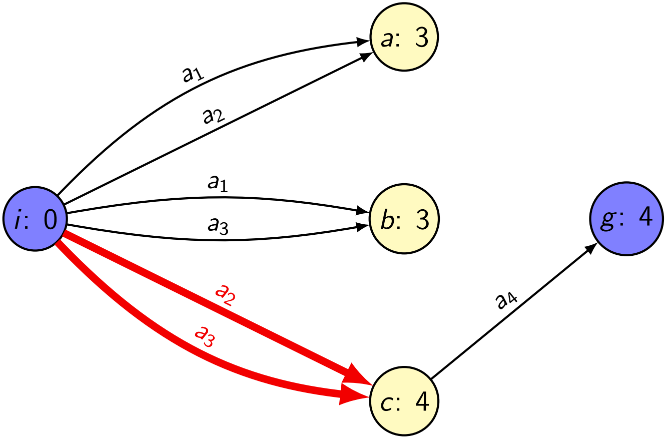
1. Compute $h^{\max}$ values of the variables.
   Stop if $h^{\max}(g) = 0$.
2. Let $P$ be a pcf that chooses preconditions with maximal $h^{\max}$ value.
3. Compute the justification graph for $P$.
4. Compute a cut which guarantees $cost(L) > 0$ for the corresponding landmark $L$.
5. Increase $h^{\text{LM-cut}}(I)$ by $cost(L)$.
6. Decrease $cost(a)$ by $cost(L)$ for all $a \in L$.

# Example: Computation of LM-Cut

## Example

round 1: $P(a_4) = c \rightsquigarrow L = \{a_2, a_3\}$ [4]

$a_1 = \langle i \to a, b \rangle_3$
$a_2 = \langle i \to a, c \rangle_4$
$a_3 = \langle i \to b, c \rangle_5$
$a_4 = \langle a, b, c \to g \rangle_0$



# Example: Computation of LM-Cut

**Example**

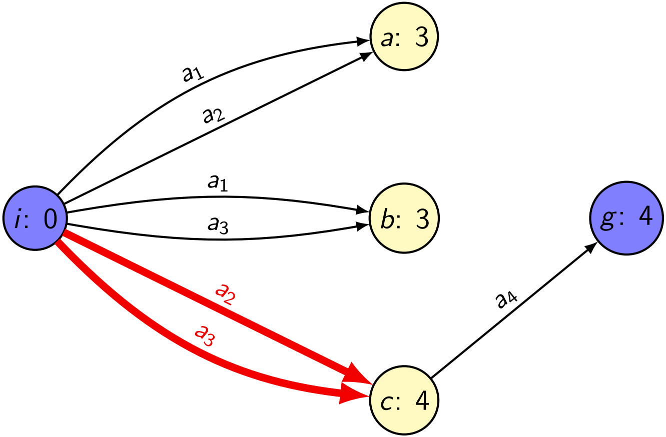
round 1: $P(a_4) = c \rightsquigarrow L = \{a_2, a_3\}$ [4] $\rightsquigarrow h^{\text{LM-cut}}(I) := 4$

$a_1 = \langle i \to a, b \rangle_3$

$a_2 = \langle i \to a, c \rangle_0$

$a_3 = \langle i \to b, c \rangle_1$

$a_4 = \langle a, b, c \to g \rangle_0$

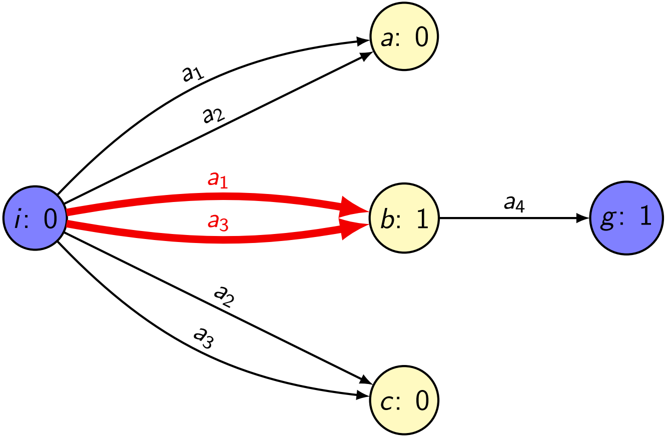


# Example: Computation of LM-Cut

## Example

round 2: $P(a_4) = b \rightsquigarrow L = \{a_1, a_3\}$ [1]

$a_1 = \langle i \to a, b \rangle_3$
$a_2 = \langle i \to a, c \rangle_0$
$a_3 = \langle i \to b, c \rangle_1$
$a_4 = \langle a, b, c \to g \rangle_0$

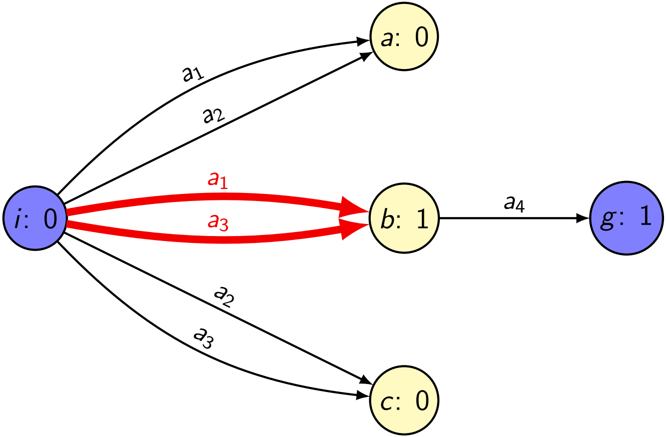


# Example: Computation of LM-Cut

## Example

round 2: $P(a_4) = b \rightsquigarrow L = \{a_1, a_3\}$ [1] $\rightsquigarrow h^{\text{LM-cut}}(I) := 4 + 1 = 5$

$a_1 = \langle i \to a, b \rangle_2$
$a_2 = \langle i \to a, c \rangle_0$
$a_3 = \langle i \to b, c \rangle_0$
$a_4 = \langle a, b, c \to g \rangle_0$

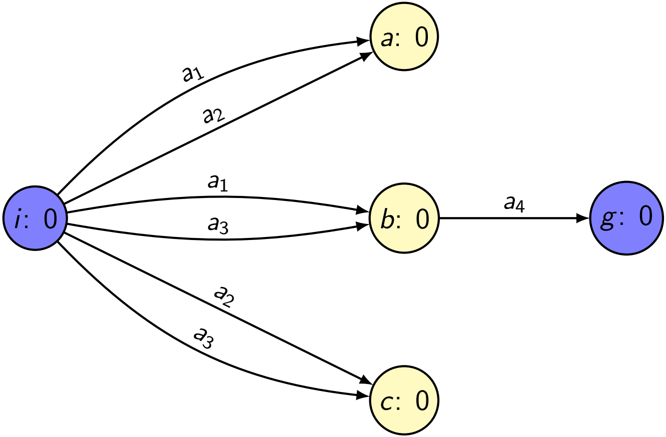


# Example: Computation of LM-Cut

## Example

round 3: $h^{\max}(g) = 0 \rightsquigarrow$ done! $\rightsquigarrow h^{\text{LM-cut}}(I) = 5$

$a_1 = \langle i \to a, b \rangle_2$
$a_2 = \langle i \to a, c \rangle_0$
$a_3 = \langle i \to b, c \rangle_0$
$a_4 = \langle a, b, c \to g \rangle_0$



# Summary

# Summary

- Cuts in justification graphs
  are a general method to find landmarks.
- Hitting sets over all cut landmarks yield a perfect heuristic
  for delete-free planning tasks.
- The LM-cut heuristic is an admissible heuristic
  based on these ideas.