

Foundations of Artificial Intelligence

21. Combinatorial Optimization: Advanced Techniques

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21.1 Dealing with Local Optima

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Combinatorial Optimization: Overview

Chapter overview: combinatorial optimization

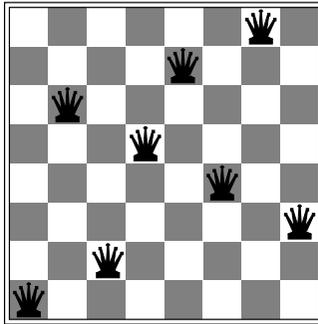
- ▶ 20. Introduction and Hill-Climbing
- ▶ 21. **Advanced Techniques**

21.1 Dealing with Local Optima

Example: Local Minimum in the 8 Queens Problem

local minimum:

- ▶ candidate has 1 conflict
- ▶ all neighbors have at least 2



Weaknesses of Local Search Algorithms

difficult situations for hill climbing:

- ▶ **local optima**: all neighbors worse than current candidate
- ▶ **plateaus**: many neighbors equally good as current candidate; none better

German: lokale Optima, Plateaus

consequence:

- ▶ algorithm gets stuck at current candidate

Combating Local Optima

possible remedies to combat local optima:

- ▶ allow **stagnation** (steps without improvement)
- ▶ include **random aspects** in the **search neighborhood**
- ▶ (sometimes) make **random** steps
- ▶ **breadth-first search** to better candidate
- ▶ **restarts** (with new random initial candidate)

Allowing Stagnation

allowing stagnation:

- ▶ do not terminate when no neighbor is an improvement
- ▶ limit number of steps to guarantee termination
- ▶ at end, return best visited candidate
 - ▶ pure search problems: terminate as soon as solution found

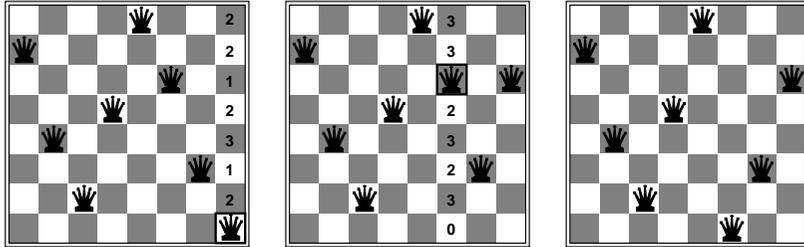
Example 8 queens problem:

- ▶ with a bound of 100 steps solution found in **94%** of the cases
- ▶ on average 21 steps until solution found
- ↪ works very well for this problem; for more difficult problems often not good enough

Random Aspects in the Search Neighborhood

a possible variation of hill climbing for 8 queens:

Randomly select a file; move queen in this file to square with minimal number of conflicts (null move possible).



↪ Good local search approaches often combine **randomness** (exploration) with **heuristic guidance** (exploitation).

German: Exploration, Exploitation

21.2 Outlook: Simulated Annealing

Simulated Annealing

Simulated annealing is a local search algorithm that systematically injects **noise**, beginning with high noise, then lowering it over time.

- ▶ walk with fixed number of steps N (variations possible)
- ▶ initially it is “hot”, and the walk is mostly random
- ▶ over time temperature drops (controlled by a **schedule**)
- ▶ as it gets colder, moves to worse neighbors become less likely

very successful in some applications, e.g., VLSI layout

German: simulierte Abkühlung, Rauschen

Simulated Annealing: Pseudo-Code

Simulated Annealing (for Maximization Problems)

$curr :=$ a random candidate

$best :=$ none

for each $t \in \{1, \dots, N\}$:

if $is_solution(curr)$ **and** ($best$ is none or $v(curr) > v(best)$):

$best := curr$

$T :=$ $schedule(t)$

$next :=$ a random neighbor of $curr$

$\Delta E := h(next) - h(curr)$

if $\Delta E \geq 0$ **or** with probability $e^{\frac{\Delta E}{T}}$:

$curr := next$

return $best$

21.3 Outlook: Genetic Algorithms

Genetic Algorithms

Evolution often finds good solutions.

idea: simulate evolution by **selection**, **crossover** and **mutation** of individuals

ingredients:

- ▶ encode each candidate as a string of symbols (**genome**)
- ▶ **fitness function:** evaluates strength of candidates (= heuristic)
- ▶ **population** of k (e.g. 10–1000) **individuals** (candidates)

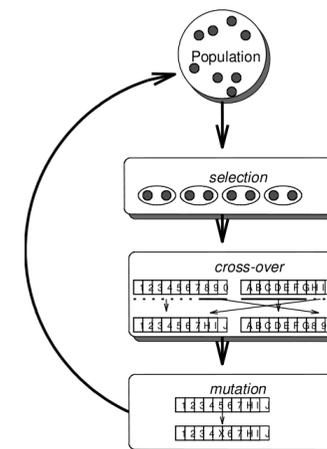
German: Evolution, Selektion, Kreuzung, Mutation, Genom, Fitnessfunktion, Population, Individuen

Genetic Algorithm: Example

example 8 queens problem:

- ▶ **genome:** encode candidate as string of 8 numbers
- ▶ **fitness:** number of non-attacking queen pairs
- ▶ use population of 100 candidates

Selection, Mutation and Crossover



many variants:

How to select?

How to perform crossover?

How to mutate?

select according to fitness function, followed by pairing

determine crossover points, then recombine

mutation: randomly modify each string position with a certain probability

21.4 Summary

Summary

- ▶ weakness of local search: **local optima** and **plateaus**
- ▶ remedy: balance **exploration** against **exploitation** (e.g., with **randomness** and **restarts**)
- ▶ **simulated annealing** and **genetic algorithms** are more complex search algorithms using the typical ideas of local search (randomization, keeping promising candidates)