Foundations of Artificial Intelligence

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Exercise Sheet 9

Due: May 9, 2016

The due date is Monday, May 9 instead of Friday, May 6 due to the holiday on May 5 and 6.

Exercise 9.1 (1.5+1.5 marks)

Consider the following syntax extension for propositional logic: $\psi \leftrightarrow \eta$ is a propositional formula if ψ and η are propositional formulas. Furthermore, let the semantics of $\leftrightarrow$ be defined by

$$I \models \psi \leftrightarrow \eta \text{ iff } I \models \psi \rightarrow \eta \text{ and } I \models \eta \rightarrow \psi.$$

Now consider the propositional formula

$$\varphi = ((A \vee B) \rightarrow C) \wedge \neg((C \vee B) \leftrightarrow A)$$

and the interpretation

$$I = \{A \mapsto \mathbf{F}, B \mapsto \mathbf{T}, C \mapsto \mathbf{T}\}.$$

(a) Use the definition of $\models$ to show that $I \models \varphi$.

(b) Create a truth table for φ. Determine if φ is valid, satisfiable, and/or falsifiable. Justify your answers.

Exercise 9.2 (1+2+3 marks)

Consider the following statements:

- "If it is raining, it is cold."
- "If it is not raining, it is not cold and the sun shines."
- "If it is not raining or if the sun is shining, Anna is in the mood for ice cream."
- "If Anna is in the mood for ice cream, she is having ice cream."

(a) Formalize the statements in propositional logic. Use proposition names that clearly express what they encode.

(b) Compile your result of Exercise 9.2 (a) to CNF by applying the logical equivalencies from the lecture (print version of Chapter 29, slide 24) and additionally commutativity (1) and associativity (2):

$$\varphi \wedge \psi \equiv \psi \wedge \varphi \text{ and } \varphi \vee \psi \equiv \psi \vee \varphi \quad (1)$$

$$((\varphi \wedge \psi) \wedge \eta) \equiv (\varphi \wedge (\psi \wedge \eta)) \text{ and } ((\varphi \vee \psi) \vee \eta) \equiv (\varphi \vee (\psi \vee \eta)). \quad (2)$$

Provide all intermediate formulas that result from applying an equivalance transformation.

(c) In the lecture, we have shown that the resolution method can be used for reasoning by a reduction to testing unsatisfiability. In this way, use the resolution method to show that the statement "It is cold or Anna has ice cream" follows logically from the given statements. Compare the number of required resolution steps with the number of entries that would have been necessary to show the same result with a truth table.

Exercise 9.3 (1.5+1.5 marks)

Use DPLL to show that the following sets of clauses are satisfiable or unsatisfiable. Provide the same intermediate results and applied rules of DPLL that were also given in the example in the lecture.

(a) $\{\{P, \neg Q\}, \{\neg P, Q\}, \{Q, \neg R\}, \{S\}, \{\neg S, \neg Q, \neg R\}, \{S, R\}\}$

(b) $\{\{P, Q, S, T\}, \{P, S, \neg T\}, \{Q, \neg S, T\}, \{P, \neg S, \neg T\}, \{P, \neg Q\}, \{\neg R, \neg P\}, \{R\}\}$

The exercise sheets can be submitted in groups of two students. Please provide both student names on the submission.