Foundations of Artificial Intelligence

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Spring Term 2016

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Exercise Sheet 8

Due: April 29, 2016

Exercise 8.1 (1+4 marks)

Consider the constraint network $\mathcal{C} = \langle V, \text{dom}, (R_{uv})\rangle$ with $V = \{A, B, C, D, E, F, G\}$, $\text{dom}(v) = \{1, \dots, 10\}$ for all $v \in V$ and $R_{uv} = \text{dom}(u) \times \text{dom}(v)$ for all pairs $u, v \in V$ except for:

$$R_{AC} = \{\langle x, y\rangle \in \text{dom}(A) \times \text{dom}(C) \mid x \leq y\}$$
$$R_{BG} = \{\langle x, y\rangle \in \text{dom}(B) \times \text{dom}(G) \mid x < y\}$$
$$R_{CE} = \{\langle x, y\rangle \in \text{dom}(C) \times \text{dom}(E) \mid 2x < y\}$$
$$R_{CG} = \{\langle x, y\rangle \in \text{dom}(C) \times \text{dom}(G) \mid |x - y| > 3\}$$
$$R_{DF} = \{\langle x, y\rangle \in \text{dom}(D) \times \text{dom}(F) \mid |y - x^2| \leq 1\}$$
$$R_{FG} = \{\langle x, y\rangle \in \text{dom}(F) \times \text{dom}(G) \mid |x - y| \leq 2\}$$

(a) Determine the constraint graph of $\mathcal{C}$.

(b) Apply the algorithm for trees as constraint graphs (slide 12 of chapter 27 of the print version of the lecture slides) to solve $\mathcal{C}$. Provide the following:

- the ordered tree, rooted at A;
- the variable ordering that is given by the ordered tree;
- the sequence of calls of the `revise` method and the resulting variable domains; and
- the result of a backtracking search with a value ordering that prefers smaller values. It suffices to provide the final variable assignments (it is, in particular, not necessary to provide the complete search tree).

Exercise 8.2 (4 marks)

Let $\mathcal{C}$ be a solvable constraint network with acyclic constraint graph. Show that the application of the algorithm for trees as constraint graphs (slide 12 of chapter 27 of the print version of the lecture slides) leads to a solution for $\mathcal{C}$ and that the algorithm does never change the values of assigned variables during application (i.e., the domain of each variable is always non-empty and the partial assignments that are considered are always consistent).

Exercise 8.3 (2+1 marks)

Consider the constraint network that is given by the graph coloring problem of a graph $G = \langle V, E\rangle$. The set of vertices V contains a vertex for each Swiss canton, and E is such that two vertices v and v' are connected iff the cantons v and v' share a border. A description of G can be downloaded from the website of the course.

(a) Provide a cutset $V' \subseteq V$ for G that is as small as possible (it is not necessary to provide an explanation how you have found V'). As a reminder, a cutset of a graph is defined as a set of vertices that is such that the induced subgraph that is obtained by removing these vertices results in an acyclic graph.

Note: You get 2 marks for your solution if your cutset is optimal, 1 mark if your cutset contains exactly one more vertex than an optimal cutset and 0 marks otherwise.

(b) Assume we are interested in coloring G with 4 colors. Provide a worst-case estimate of the runtime of the algorithm based on cutset conditioning if your cutset of exercise 8.3 (a) is used (i.e., compute an upper bound for the number of considered assignments). Compare your result to the estimated runtime if no cutset is used.

Hint: It is simpler to solve this exercise if a graph visualization tool like, for instance, *graphviz* is used. The description of G that can found on the website of the course is formatted for graphviz. If you use Linux, you can create a pdf visualization of the graph with:

```
dot -T pdf -o cantons.pdf cantons.dot
```

The exercise sheets can be submitted in groups of two students. Please provide both student names on the submission.