Foundations of Artificial Intelligence

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Exercise Sheet 4

Due: April 1, 2016

Exercise 4.1 (2+1.5+1+1.5 marks)

Consider the 8-puzzle (a problem that is analogous to the 15-puzzle that was introduced in the lecture) with the following initial state:

1	2	3
4	5	
7	8	6

The available actions are the same as in the 15-puzzle: The blank can be moved *up*, *left*, *right* and *down* unless it is already at the corresponding border. The problem's only goal state is the following state:

1	2	3
4	5	6
7	8	

(a) Apply depth-limited search with a depth limit of 3 to the initial state (i.e., simulate a call to `depth_limited_search(init(), 3)`). Provide the resulting search tree, including the order in which search nodes are expanded. The order in which successor nodes are generated is given by the action order *up* < *left* < *right* < *down* (i.e,, the first successor that is generated for each state is the state that results from moving the blank *up* and the last one is the successor where the blank is moved *down*).

(b) What is the result of applying depth-first search (without depth limit)? Justify your answer.

(c) Consider the same scenario as in a), but with reversed action order (i.e., *down* < *right* < *left* < *up*). Simulate a call to `depth_limited_search(init(), 3)` with the altered action order and compare the result to the result of a).

(d) Assume that both iterative deepening search and breadth-first search are applied to any solvable instance of the 8-puzzle. Compare the asymptotic behavior of both algorithms by comparing both algorithms with respect to the expected runtime and the expected number of node expansions until a solution is found. (Note that it is not necessary to provide exact numbers here, a justified, relative statement suffices.)

Exercise 4.2 (2+2+2 marks)

(a) Consider the heuristic h_1 for the "missionaries and cannibals" problem where

$$h_1(\langle m, c, b\rangle) := \max\{m + c - b, 0\}.$$

As a quick reminder: states are triple $\langle m, c, b\rangle \in \{0, 1, 2, 3\} \times \{0, 1, 2, 3\} \times \{0, 1\}$, where m gives the number of missionaries, c the number of cannibals and b the number of boats that are at the *wrong* river bank in the evaluated state. Keep in mind that the boat can carry no more than two persons.

Determine if h_1 is *safe*, *goal-aware*, *admissible* and/or *consistent*. Justify your answer for each property.

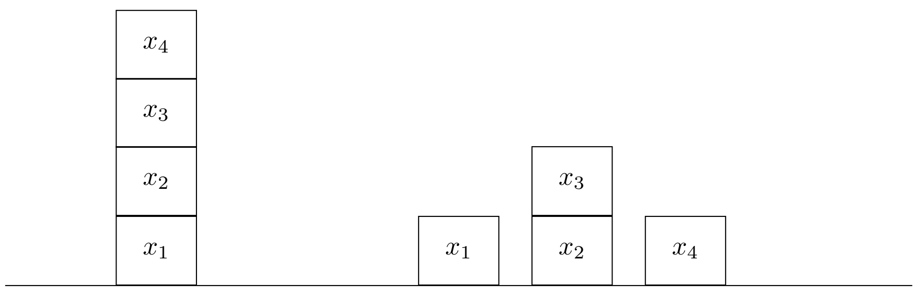
(b) Let $x_1, \dots, x_n$ denote the blocks in a blocks world problem. Consider the heuristic

$$h_2(s) := \sum_{i=1}^{n} f(x_i)$$

for blocks world, where the function f is defined as:

$$f(x_i) := \begin{cases} 1 + |\{x_j \mid x_j \text{ is anywhere above } x_i\}| & \text{if } \mathit{goalpos}(x_i) \neq \mathit{pos}(s, x_i) \\ 0 & \text{otherwise.} \end{cases}$$

The expression $\mathit{goalpos}(x_i) \neq \mathit{pos}(s, x_i)$ holds if block x_i is on block y in state s, but should be on block $z \neq y$ in the goal (as discussed in the lecture, y and z may also represent the table). The heuristic h_2 hence determines all blocks that are not yet at their goal position and adds all blocks above those blocks (since they have to be moved before those blocks can be moved). To illustrate the heuristic, consider the following example:



The initial state s_0 is depicted on the left side, and the goal state on the right. The heuristic value in the initial state is $h_2(s_0) = 0 + 3 + 0 + 1 = 4$ as x_1 and x_3 are on the correct block already and hence $f(x_1) = f(x_3) = 0$. Block x_2 is on x_1 instead of the table with 2 blocks above x_2, so $f(x_2) = 3$, and block x_4 is on x_3 instead of the table and there are no blocks above x_4, so $f(x_4) = 1$.

Determine if h_2 is *safe*, *goal-aware*, *admissible* and/or *consistent*. Justify your answer for each property.

(c) At least one of the heuristics h_1 and h_2 is not *safe*, *goal-aware*, *admissible* **and** *consistent* (i.e., at least one of these properties does not hold for at least one of the heuristics). For one of the problems where this holds, give an alternative, non-trivial heuristic that incorporates all four of these properties (your heuristic should be tailored to the problem and use specific state information; the blind heuristic, for instance, which assigns a value of 0 to all goal states and of 1 to all other states is considered a trivial heuristic for this exercise).

The exercise sheets can be submitted in groups of two students. Please provide both student names on the submission.