Theory of Computer Science

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Exercise Sheet 11

Due: Wednesday, May 13, 2015

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Exercise 11.1 (Decidability and Semi-Decidability, 0.5+0.5+1+1+1 Points)

Which of the following statements are true, which are false? In each case, specify a short proof (1 sentence) or a counter example. You can use all results from the lecture.

(a) Every decidable language is of type 0.

(b) If A is decidable then $\bar{A}$ is also decidable.

(c) Every language that is accepted by a Turing machine is decidable.

(d) Every language that can be described by a regular expression is decidable.

(e) Every decidable language is context-free.

Exercise 11.2 (Transitivity of Reductions, 1 Point)

Show for any languages A, B and C: if $A \leq B$ and $B \leq C$, then $A \leq C$.

Exercise 11.3 (Emptiness Problem, 2 Points)

The *emptiness problem* for general grammars (i.e. type-0 grammars) is defined as:

$$\textsc{Emptiness}: \text{ Given a general grammar } G, \text{ is } \mathcal{L}(G) = \emptyset?$$

Prove that Emptiness is undecidable.

Hints: you can use without proof that there is a computable function that transforms a given type-0 grammar G to a DTM M_G with $\mathcal{L}(M_G) = \mathcal{L}(G)$. Likewise, there is a computable function that transforms a given DTM M to a type-0 grammar G_M with $\mathcal{L}(M) = \mathcal{L}(G_M)$.
Use Rice's theorem in an appropriate way to show the undecidability.

Exercise 11.4 (Undecidable Grammar Problems, 1.5+1.5 Points)

The *equivalence problem* and the *intersection problem* for general grammars are defined as:

- Equivalence: Given two general grammars G_1 and G_2, is $\mathcal{L}(G_1) = \mathcal{L}(G_2)$?
- Intersection: Given two general grammars G_1 and G_2, is $\mathcal{L}(G_1) \cap \mathcal{L}(G_2) = \emptyset$?

(a) Show that Equivalence is undecidable, by reducing Emptiness to it.

(b) Show that Intersection is undecidable, by reducing Emptiness to it.

Hint: of course you can use the fact that Emptiness is undecidable even if you did not complete exercise 11.3.

Exercise 11.5 (Rice's Theorem, 1 Bonus Point)

For which of the following languages does Rice's theorem show that the language is undecidable? For each language where Rice's theorem can be used, specify the subset of Turing-computable functions $\mathcal{S}$ for which you use the theorem.

Hint: You do not have to write down any proofs. If Rice's theorem is applicable, specify the set $\mathcal{S}$, otherwise give a short reason (1 sentence) why Rice's theorem is not applicable.

(a) $L = \{w \in \{0,1\}^* \mid M_w \text{ does not terminate with a valid output for any input } \}$

(b) $L = \{w \in \{0,1\}^* \mid M_w \text{ computes the successor function or the predecessor function } \}$

(c) $L = \{w \in \{0,1\}^* \mid M_w \text{ requires an even number of steps on the input } \texttt{0011} \}$

(d) $L = \{w \in \{0,1\}^* \mid \text{No input of } M_w \text{ leads to a valid output containing } \texttt{0} \}$