Theory of Computer Science

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Exercise Sheet 10

Due: Wednesday, May 6, 2015

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Exercise 10.1 (μ-Operator; 2 Points)

Let P be a WHILE-program which computes $f : \mathbb{N}_0^2 \to \mathbb{N}_0$ and only utilizes variables $x_0, \dots, x_4$. Specify a WHILE-program which calculates μf. You may use all abbreviated notations from the lecture and earlier exercises.

Hint: Ensure that all variables used by P have the right initial value each time you call P, and remember that P can change these variables.

Exercise 10.2 (Enumerable Functions, 1+1.5 Points)

Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. Specify total and computable functions $f : \mathbb{N}_0 \to \Sigma^*$ which recursively enumerate the following languages. Additionally specify the function values $f(0)$, $f(1)$, …, $f(5)$. You may use all computable functions which were discussed in the lecture.

(a) $L_1 = \{\mathtt{b}^n\mathtt{a}^{2n} \mid n \in \mathbb{N}_0, n \text{ is even}\}$

(b) $L_2 = \{w \in \Sigma^* \mid \mathtt{a} \text{ occurs in } w \text{ exactly once}\}$

Exercise 10.3 (Recursively Enumerable Languages, 1+1.5 Points)

Specify for the following functions which language they enumerate recursively.

(a) $f(x) = \mathtt{replace}_{\mathtt{ab}}(bin(x+1))$

where $\mathtt{replace}_{\mathtt{ab}} : \{0,1\}^* \to \{\mathtt{a}, \mathtt{b}\}^*$ replaces all '0'es by $\mathtt{a}$ and all '1's by $\mathtt{b}$.

(b) $f(x) = \begin{cases} f_A(\frac{x}{2}) & \text{if x is even} \\ f_B(\frac{x-1}{2}) & \text{otherwise} \end{cases}$

where $f_A(x)$ and $f_B(x)$ are total and computable functions which recursively enumerate the langauge A respectively the language B.

Exercise 10.4 (Decidability and Semi-Decidability, 1+1+1 Points)

(a) Prove that if $A \subseteq \Sigma^*$ and $B \subseteq \Sigma^*$ are semi-decidable, then $A \cup B$ is semi-decidable.

(b) Prove that if $A \subseteq \Sigma^*$ and $B \subseteq \Sigma^*$ are decidable, then $A \cap B$ is decidable.

(c) Proposition: The following algorithm calculates $\chi'_{A \cup B}$ (where A and B are semi-decidable):

```
INPUT: w
IF χ'_A(w) = 1 THEN
  RETURN 1
ELSE IF χ'_B(w) = 1 THEN
  RETURN 1
ELSE
  LOOP FOREVER
```

Why is this proposition not true in the general case?