Theory of Computer Science

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Exercise Sheet 9

Due: Wednesday, April 29, 2015

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Exercise 9.1 (Modified Subtraction; 1 Point)

Prove that the following definition of the modified substraction is correct, that is $sub(x, y) = \max(x - y, 0)$ for all $x, y \in \mathbb{N}_0$. Also specify definitions of h and sub_{rev} in common mathematical notation.

$$\begin{aligned} h &= substitute(3, 1, pred, \pi_1^3) \\ sub_{rev} &= primitive\_recursion(\pi_1^1, h) \\ sub &= substitute(2, 2, sub_{rev}, \pi_2^2, \pi_1^2) \end{aligned}$$

Exercise 9.2 (Primitive-Recursive Functions; 4 Points)

On the lecture website you will find a Java program where you can define and evaluate primitive-recursive functions. Define the following functions with the help of this program, but without using the μ-operator (thus showing that the functions are primitive-recursive).
In your solution you may use all functions which were defined in the lecture. The file `lecture.def` contains their definitions. Add your definitions at the end of this file below the corresponding marker. Any changes above the marker will be ignored.
In each case, demonstrate the correctness of your definition for some examples using the `print` function.

(a) $add\_succ : \mathbb{N}_0^2 \to \mathbb{N}_0$ with $add\_succ(x, y) = x + y + 1$ for all $x, y \in \mathbb{N}_0$.

(b) $binom_2 : \mathbb{N}_0 \to \mathbb{N}_0$ with $binom_2(x) = \binom{x}{2}$ for all $x \in \mathbb{N}_0$

You can use without proof that for all $n \in \mathbb{N}_0$: $\sum_{i=1}^{n} i = \binom{n+1}{2}$

(c) $encode : \mathbb{N}_0^2 \to \mathbb{N}_0$ with $encode(x, y) = \binom{x+y+1}{2} + x$ for all $x, y \in \mathbb{N}_0$.

(d) $pow : \mathbb{N}_0^2 \to \mathbb{N}_0$ with $pow(x, y) = x^y$ for all $x, y \in \mathbb{N}_0$.

Exercise 9.3 (μ-Operator; 3 Points)

For each of the following functions f specify a definition of μf in common mathematical notation and prove that your definition is equivalent to μf.

(a) $f(x, y, z) = z \ominus y^x$ for all $x, y, z \in \mathbb{N}_0$

(b) $f(x, y, z) = (y \ominus x) \cdot (z \ominus x)$ for all $x, y, z \in \mathbb{N}_0$

(c) $f(x, y, z) = (y \ominus x) + (z \ominus x)$ for all $x, y, z \in \mathbb{N}_0$

Exercise 9.4 (μ-rekursive Functions; 2 Points)

Show with the program from exercise 9.2 that the function $f : \mathbb{N}_0 \to \mathbb{N}_0$ with $f(x) = \lceil \sqrt[3]{x} \rceil + 1$ for all $x \in \mathbb{N}_0$ is μ-recursive. You may use all μ-recursive functions shown in the lecture. Demonstrate the correctness of your definition for some examples using the `print` function.