Theory of Computer Science

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Exercise Sheet 4

Due: Wednesday, March 25, 2015

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Exercise 4.1 (Predicate logic; 1 Point)

Consider the formula φ over a signature with predicate symbols P (1-ary), Q (2-ary) and R (3-ary), the 1-ary function symbol f, the constant symbol c and the variable symbols x, y and z.

$$\varphi = (\forall x \exists y\, (\mathrm{P}(z) \to \mathrm{Q}(y, x)) \vee \neg \exists y\, \mathrm{R}(\mathrm{c}, x, \mathrm{f}(y)))$$

Mark all occurrences of free variables in φ. *Additionaly* specify the set of free variables of φ (without proof).

Exercise 4.2 (Formal grammars; 0.5 + 2 + 0.5 Points)

Consider the following formal language:

$$S = \{a^n b^{2n} \mid n \geq 0\}$$

(a) Is ε an element of S? Justify your answer.

(b) Specify a *complete description* of a formal grammar G that generates S (i.e., $\mathcal{L}(G) = S$). A formal grammar is a four tuple $G = (\Sigma, V, P, S)$, remember to define all components of this tuple.

(c) Which types (in the Chomsky-Hierarchy) is your formal grammar part of? You don't have to prove your answers.

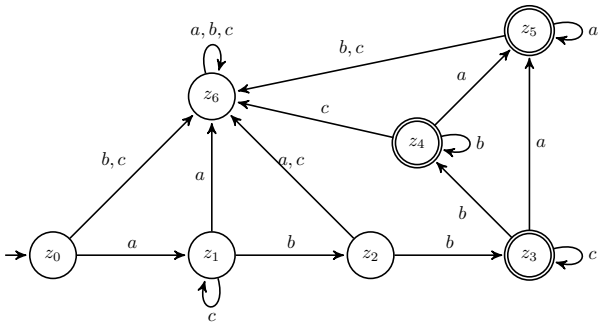
Exercise 4.3 (Derivation of words; 1 Points)

Consider the following formal grammar $G = (\Sigma, V, P, E)$ with $\Sigma = \{0, 1,), (, +, -\}$, $V = \{E, A, M, Z\}$ and and the following rules in the set P:

$E \to A$	$E \to M$	$E \to 0$	$E \to 1Z$	
$A \to (E + E)$	$M \to (E - E)$	$Z \to 0Z$	$Z \to 1Z$	$Z \to \varepsilon$

Specify a derivation of the word "$((10 + 1) - (0 - 101))$".

Exercise 4.4 (DFA and regular grammar; 1.5 + 1.5 Points)
Consider the following DFA M:



(a) Which language does the DFA accept?

(b) Specify a *regular* grammar, which produces the same language.

Exercise 4.5 (DFA; 2 Points)
Specify a deterministic finite automaton that accepts the language over $\Sigma = \{a, b\}$. The words of the language have following property.
When a occurs at the beginning of the word or after b, then there directly follows at most one further a. When b occurs at the beginning of the word or after a, then there directly follows at least one further b.