Theory of Computer Science

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Exercise Sheet 3

Due: Wednesday, March 18, 2015

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Exercise 3.1 (Refutation Theorem; 2 Points)

Prove the refutation theorem, that is, show for any set of formulas KB and any formula φ that

$$\mathrm{KB} \cup \{\varphi\} \text{ is unsatisfiable if and only if } \mathrm{KB} \models \neg\varphi.$$

Exercise 3.2 (Inference; 1+1+1+1 Points + 1 Bonus Point)

You'll find a Java program on the lecture website that checks proofs formulated in propositional logic. Use this program to prove following statements. For a statement of the form WB $\models \varphi$ write a text file containing a derivation that only uses formulas from WB as assumptions and that has φ in its last line. An example for this is contained in the file `proof.txt`.

(a) $\{A, B\} \models ((A \wedge B) \vee C)$

(b) $\{(A \wedge B)\} \models (A \rightarrow (B \vee C))$

(c) $\{((A \vee B) \rightarrow (A \rightarrow C)), A\} \models C$

(d) $\{((C \vee D) \leftrightarrow (A \wedge B)), \neg E, (((A \wedge B) \wedge (C \vee D)) \rightarrow E)\} \models \neg(A \wedge B)$
For this exercise, extend the calculus by a new rule *negation-introduction*:

$$\frac{(\varphi \rightarrow \psi), (\varphi \rightarrow \neg\psi)}{\neg\varphi}$$

(e) *Bonus exercise:* To show that a calculus is correct, we have to prove that all rules are correct. Show the correctness of the rule *negation-introduction*

Note on the submission process: please create one text file for each exercise part which contains the derivation. The program must be able to parse the file and accept the derivation as correct. The new rule (*negation-introduction*) requires a new line in the program. Copy this line on your regular submission. The bonus exercise cannot be solved with the program.

Exercise 3.3 (Resolution Calculus; 2 Points)

Consider the following knowledge base

$$\text{KB} = \{(A \leftrightarrow \neg D), (\neg A \rightarrow (B \vee C)), ((A \rightarrow E) \wedge (B \vee C \vee F)), (E \rightarrow (F \rightarrow (B \vee C))), (C \rightarrow G), (G \rightarrow \neg C)\}.$$

Use the resolution calculus to show that $\text{KB} \models (B \wedge \neg C)$.
Note: A proof using resolution consists of three steps (see lecture slides for an example). Use the notation from the lecture slides in particular in the last step, that is, use one line for each derived clause together with the derivation's justification. Schöning also uses this notation for the third step on page 36 (the first part of the example, not the visualization).

Exercise 3.4 (Predicate Logic; 2 Points)

Consider the following predicate logic formula φ with the signature $\langle\{x, y\}, \{\mathrm{c}\}, \{\mathrm{f}, \mathrm{g}\}, \{\mathrm{P}\}\rangle$.

$$\varphi = (\neg \mathrm{P}(\mathrm{c}) \wedge \forall x \exists y\, ((\mathrm{f}(y) = \mathrm{g}(x)) \wedge \mathrm{P}(y)))$$

Specify a model $\mathcal{I}$ of φ with $\mathcal{I} = \langle U, \cdot^{\mathcal{I}} \rangle$ and $\mathcal{U} = \{u_1, u_2, u_3\}$. Prove that $\mathcal{I} \models \varphi$. Why is no variable assignment α required to specify a model of φ?